ISSN (Print): 2476-8316
ISSN (Online): 2635-3490
Dutse Journal of Pure and Applied Sciences (DUJOPAS), *Vol. 6 No. 3 September 2020*

Formulating States and Transitions for Shiva Family *DARA* Game

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Abstract
*The Shiva family Dara game (Dara) has 5 x 6 board size with tree-search theoretical structure, thus it could be modeled as an Artificial Intelligence (AI) search problem so as to yield a new test-bed for evaluating search algorithms, and also to support building of AI agent for playing Dara. For successful modeling of Dara as an AI search problem, it is necessary to formulate States and Transition function for the game. This paper formulates a mathematical model for the states and transition function of Dara.The work provides mathematical representations of 3.684 * 10^{29} legal and illegal states of Dara game, which will serve as input in developing the computer version of the game.*

Keywords: Algorithm, Dara, game state, transition function

INTRODUCTION
Besides the traditional purpose of deriving fun and entertainment, some games such as Chess and Go have been formulated as Artificial Intelligence (AI) search problems. Such models of games are widely used as test-bed for evaluating AI search algorithms such as Minmax and MCTS. Thus, leading to development of AI solutions and techniques, which are applicable in addressing relevant real-world problems. The applications of games span areas like security and networking among others (Horak, Bosanky and Pechoucek, 2017). Games are categorized into combinatorial games, Real Time Strategy (RTS) games, etc, based on four criteria, namely: Goal, Number of players, Turn based, and Information availability. This research focuses on a combinatorial game called Dara. Combinatorial games are zero-sum board games with perfect

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information where two players alternate moves (Garg, Songara and Maheshwari, 2017; Victor and Chaimowicz, 2017). Examples of combinatorial games include popular games like Go, chess, checkers, Tic-tac-toe, and so on.

Dara is a combinatorial game that falls under the Shiva family of games. Dara is a board game with two players aiming at having three of their playing pieces arranged orthogonally. Dara ends when any of the two players captures ten playing pieces of the opponent player, or blocks all opponents'liberties (Zaslavsky, 1998).

In order to model the Dara game as an AI search problem, the game must be defined as five tuples, namely: the initial state of the game, set of operators/actions, transition function, set of final states, and the path cost. The paper aims at describing how States and Transition function could be formulated for the game of *Dara.* Modeling the Dara game as an AI search problem was presented in our earlier work (AbdulHakim and Umar, 2019).

Mathematical formulation was adopted to represent some special patterns in the game of Go, due to the theoretical nature of Go (Sato, Anada and Tsutsumi, 2016; Kim, 2017). This was the same reason we consider adopting mathematical formulation in coming up with the proposed formulation for the states of *Dara* game. Garg *et al.* (2017) use theoretical computer science to proof a winning strategy for the game of Tic Tac Toe, which is also among the three in row games like Dara. However, Dara game does not terminate with the first three-in-row, which is the case in Tic Tac Toe. In the literature, no work on formulating DARA game as an AI search problem was found. Consequently, no formulation for states and transitions was proposed in the literature. Our earlier paper (AbdulHakim and Umar, 2019) proposes a formal search problem formulation for Dara game.

This article provides detail mathematical formulation of states in*Dara* game and transitions among the states, which serve as one of the five tuples of AI search problem model(Yang *et al.*, 2017).

Modeled Dara Game

In an earlier paper titled "Formulation of Shiva Dara game as a test-bed for evaluating the performance of AI search algorithms" (AbdulHakim and Umar, 2019), Dara game was modeled as an AI search problem defined using five tuples listed below (Russell and Norvig, 2009)

- S_0 is the initial state of the game
- $\sum$ is the set of operators/actions
- ∂ is the transition function (∂=QX$\sum$)
- F is the set of final states
- c is the path cost.

The focus of this paper is a detailed presentation of how states and transition functions are formulated.

Formulation of Game States and Transitions

Considering the size complexity of the board of Dara game, there are approximately $3.684 * 10^{29}$ possible ways of positioning the twenty-four playing pieces on the board of Dara game, this gives the possible number of states or configurations of playing pieces on the board. We should note that not all the configurations make legal state in Dara. The set of legal states in the Dara game comprises all states in which no more than three pieces of the same player are arranged orthogonally (horizontally or vertically) (Erica, 2018).

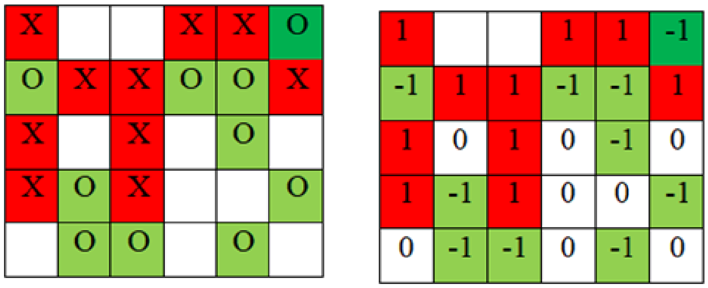
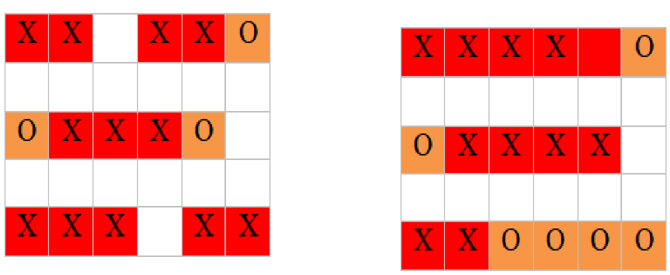


Figure 1: Accepted State and Corresponding Values of Cells

To model the acceptable states in Dara game in compliance with rules of the game, groups of all possible configurations of four (4) adjacent cells in rows and columns have been established in truth tables, and ensure that the sum of values in each group is within the range of -3 to +3. Adopting the convention of matrix notations, we assumei representrow index and j representcolumn index as illustrated in Figure 4. The sets of four consecutive cells obtainable from each row are j0 to j3, j1 to j4, j2 to j5. For each of the columns there will be i0 to i3, i1 to i4, which give two groups of four cells over columns. Hence, there are fifteen groups of four cells across the five rows and twelve groups of four cells over the six columns, which equates to twenty-seven (27) possible groups of four cells in any configuration of the board. For an acceptable state in the game of Dara, the sum of values of the cells in each group should fall within the range -3 to 3. The possible values for acceptable configurations of playing pieces including empty spaces can be verified using set of truth tables.



(a) Valid (b) Invalid

Figure 2: Sample of Configuration across rows

The sum of four cells in fifteen groups across rows is depicted in Equation 1 and a sample of valid and invalid configurations are given in Figure 2(a) and (b) respectively.

Equation 1

$$P_r = \left\{ p : \underset{i=0}{\overset{4}{\forall}} \sum_{j=0,1,2}^{j+3} x_{ij} = y \wedge -3 \leq y \leq 3 \right\}$$

Where Pr represents the set of acceptable patterns across rows.

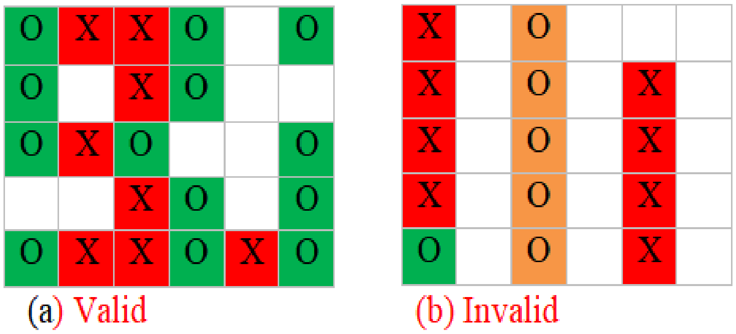


Figure 3: Sample of Config. over columns

The sum of four cells in twelve groups over the columns are depicted in Equation 2 and a sample of valid and invalid configuration of playing pieces is given in Figure 3(a) and (b) respectively.

Equation 2

$$P_c = \left\{ p : \forall_{j=0}^{5} \left(\sum_{i=0,1,}^{i+3} x_{ij} = y \right) \wedge (-3 \leq y \leq 3) \right\}$$

Where, Pc represents the set of acceptable patterns over the columns.

Merging the sets of patterns for groups in rows (Equation 1) and that in columns (Equation 2), gives the general set of valid patterns in acceptable states on the board of Dara, Equation 3 is established to depict all the legal states in the game of Dara. Equation 4 provides another way to depict the elements of the set of valid states (Q).

A state is a valid state if the configuration of playing pieces for the current player complies with acceptable patterns in rows (Pr) and columns (Pc). Equation 3 and Equation 4 represent a set of legal states.

Equation 3

$$Q_l = \left\{ q : \forall_{i=0}^{4} \forall_{j=0}^{5} \left(\sum_{i=0,1}^{i+3} \sum_{j=\theta 0,1,2}^{j+3} x_{ij} = y \right) \wedge -3 \leq y \leq 3 \right\}$$

Alternatively represented in terms of Pr and Pc in equation 4

Equation 4

$$Q_l = \left\{ q : P_c \cup P_r = 1 \right\}$$

States in Positioning Phase

This is the set of all the states in the first phase of the game, which commences with empty board (S_0) with each player being allocated with twelve pieces which are to be dropped alternatively between the two players until exhausted. In this phase, there should be no more than two pieces of the same player in consecutive cells, as stated in the rules of the game(Kabir and AbdulHakim, 2019). To represent this, we group the indices of the board into threes and ensure that the sum of each group is within the range -2 to 2 for valid states. In other words, positioning phase is a subset of the set of legal states with limits of the range exclusive.

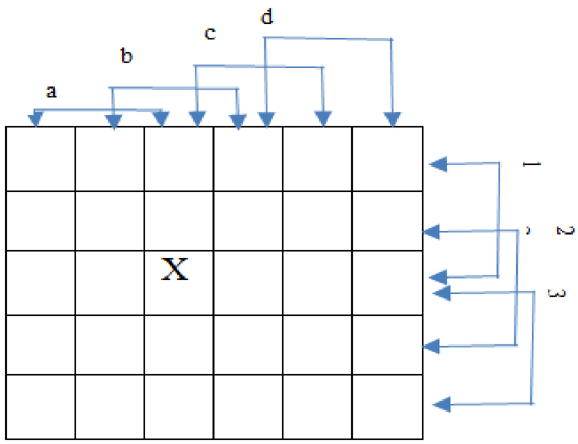


Figure 4: Grouping cells in threes

Consider Figure 4, the sets of three-in-row that can be derived from each row are a (j1 to j3), b (j2 to j4), c (j3 to j5), d (j4 to j6) which gives four groups of three cells. For each column there are 1 (i1 to i3), 2 (i2 to i4), 3 (i3 to i5) which gives three groups of threes over each column. Hence, there are twenty (20) sets of threes across the five rows and eighteen (18) groups of threes in column over the six columns, which equates to thirty-eight (38) possible groups of threes. Any sum of three consecutive cells that results in a value outside the range of -2 to 2 will lead to invalid, board configuration (state) in the positioning phase. Mathematical representation of the scenario is in Equation 5.

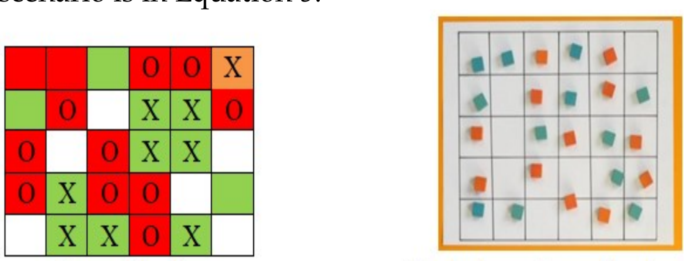


Figure 5: Sample of Valid State in Positioning

Equation 5

$$Q_p = \left\{ q : \overset{4}{\underset{i=0}{\forall}} \overset{5}{\underset{j=0}{\forall}} \left(\sum_{i=0,1,2}^{i+2} \sum_{j=0,1,2,3}^{j+2} x_{ij} = y \right) \wedge -3 < y < 3 \right\}$$

Equation 6 depicts a special state with twenty four pieces, which is a transition state between the two phases of the game.

Equation 6

$$Q_{mp} = Q_p : X_{ij} \neq 0 \wedge \sum_{i=0}^{4} \sum_{j=0}^{5} \left| X_{ij} \right| = 24$$

States in Movement Phase
This is the set of all legal states in the second phase of the game, the phase commence with the termination of positioning phase as illustrated in Equation 7. In the early stage of this phase there are six empty cells which can be used by either player, the players alternate turn by moving their playing pieces to the adjacent empty cells with the aim of arranging three pieces orthogonally.
To adhere with rule of the game, all possible groups of four adjacent cells were established. The ranges $j_0$to J_3, $j_1$to J_4,$j_2$to J_5 represent the possible combinations of four consecutive cells across the rows, and ranges $i_0$to i_3, $i_1$toi_4 represent the combination of four consecutive cells over the columns. The sum of values in each group is ensured to fall within the range of -3 to 3.
The sums of four adjacent cells in twenty groups across rows and eighteen groups over columns are depicted in Equation 7.

Equation 7

$$Q_m = \left\{ q : \mathop{\forall}_{i=0}^{4} \mathop{\forall}_{j=0}^{5} x_{ij} \left(\sum_{i=0,1,2}^{i+3} \sum_{j=0,1,2,3}^{j+3} x_{ij} = y \right) \wedge -3 \le y \le 3 \right\}$$

It should be noted that set of states in movement phase is a subset of legal states so alsoset of states in positioning phase. Hence the set of accepted states of Dara game can be represented in relation to the states discussed in the previous sections as in Equation 8.

Equation 8

$$Q = Q_l = S_0 \cup Q_p \cup Q_m \cup F$$

1.1 States Transitions

Transitional model provides a function that defines the transition from one state (Q) to its successive state (Q+) when an action element of the set is applied(Chatterjee *et al.*, 2017; Lelis, 2017; Yang *et al.*, 2017). The general transition function is in Equation 9 as adopted from literature, where Q is the set of legal states, Q+ is the successive state, and is the set of actions.

Equation 9

$$\partial : Q \times \xi \rightarrow Q_+$$

Source: (Chatterjee *et al.*, 2017; Lelis, 2017)

We further customize the general transition function to conform to some of the important stages (states transitions) in the respective phases of the game. Transition from initial state $S_0$to any state in positioning phase can only be achieved by applying positioning action to generate state Qp. The transition is depicted in Equation 10(a).

Equation 10

$$\partial : S_0 \times a_p \rightarrow Q_p \_\_\_(a)$$
$$\partial : Q_p \times a_p \rightarrow Q_p \_(b)$$
$$\partial : Q_p \times a_m \rightarrow \varnothing \_(c)$$
$$\partial : Q_p \times a_c \rightarrow \varnothing \_(d)$$

The transitions that occur in phase 1 of the game are represented in Equation 10(b), 10(c) and 10(d) show that movement, and capture action is not applicable in the first phase of the game. Equation 10(a) represents the transition between the phases, where movement action is allowed to special positioning state Q_{mp}(positioning state with twenty-four pieces). Transition among states in movement phase Equation 11(a)-11(d) is only obtainable through movement or capture action, and sometimes the result of transition leads to either state in movement phase or final state.

Equation 11

$$\partial : Q_{mp} \times a_m \rightarrow Q_m \_(a)$$
$$\partial : Q_m \times a_p \rightarrow \varnothing \_(b)$$
$$\partial : Q_m \times a_m \rightarrow Q_{2p} \_(c)$$
$$\partial : Q_m \times a_c \rightarrow Q_{2p} \_(d)$$
$$where$$
$$Q_{2p} = Q_m \cup F$$

Set of Final States

A state is the final state, if it passes the goal test or has no successor node (terminal node). Player is defeated when there is no liberty for any of his playing pieces, or when his pieces are captured that he cannot make three in either row or column. In other words, the game terminates when either player wins the game or either player loses liberty. Sample of winning states are given in Figure 6, which were depicted Equation 12 (a) through Equation 12(c).

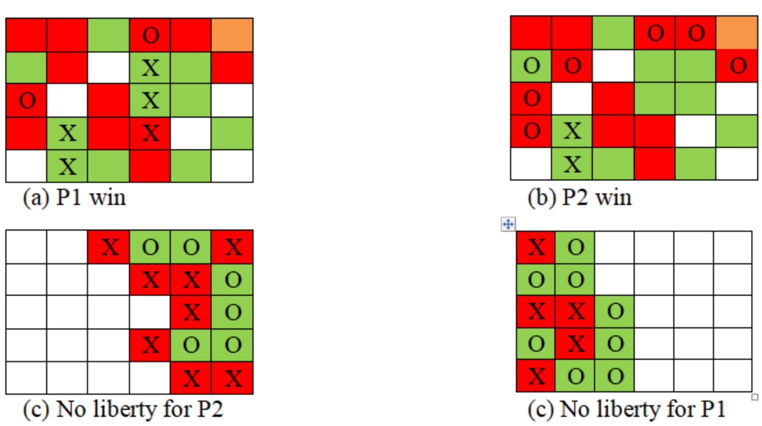


Figure 6: Sample of Final State

Equation 12

$$XWin = \left\{ s : \underset{i=0}{\overset{4}{\forall}} \underset{j=0}{\overset{5}{\forall}} x_{ij} \neq -1, \sum_{i=0}^{4} \sum_{j=0}^{5} x_{ij} \geq -2 \right\} \_\_\_\_ (a)$$

$$OWin = \left\{ s : \underset{i=0}{\overset{4}{\forall}} \underset{j=0}{\overset{5}{\forall}} x_{ij} \neq 1, \sum_{j=0}^{4} \sum_{j=5}^{5} x_{ij} \leq 2 \right\} \_\_\_\_\_\_ (b)$$

$$F = XWin \vee OWin \vee \overline{L_X} \vee \overline{L_O} \_\_\_\_ (c)$$

DISCUSSION OF RESULTS

The mathematical equations presented in the section 3 basically represent the five categories of legal states in Dara and possibletransitions among the states. The five major categories include the initial state, phases transition state (equation 6),set of final states, set of states in positioning phase and set of states in movement phase. These collectively form an element of a set of five tuple model of artificial intelligence search problem as inRussell and Norvig (2009). The representations provides an in-depth description of a component in a proposed model of authors' previous publication (AbdulHakim and Umar, 2019), which was implemented and evaluated with some proposed computer agents as published in the recent article of Kabir and AbdulHakim (2020).Zhang (2007)modeledstates and transitions for game of tic-tac-toe share similarities with the equations in section 3; both adopt mathematical representations. However, Zhang consider the problem (tic-tac-toe) as model checking problem, and our formulations consider the Dara as AI search problem.

CONCLUSION

Dara game could be modeled as an AI search problem, and thus be applied as a test-bed for evaluating AI search algorithms. Such a model for Dara game was defined as five tuples which includes States and Transition Function. This paper introduced mathematical formulations of states and transition functions for Dara games. The numerous equations for States and Transitions presented in this paper were employed in developing an AI agent for playing Dara games.

REFERENCES

AbdulHakim, I. and Umar, K. (2019) 'On the Formulation of Shiva Dara Game as a Testbed for Evaluating Artificial Intelligence Search Algorithms', in *Proceedings of the 4th YUMSCIC*, pp. 416–422.

Chatterjee, K. *et al.* (2017) 'Optimizing Expectation with Guarantees in POMDPs', in *Thirty-First AAAI Conference on Artificial Intelligence*, pp. 3725–3732. Available at: http://arxiv.org/abs/1611.08696.

Erica, E. (2018) *Dara Game: Nigerian 3 in a Row with a Twist*, *What Do we DO All Day*. Available at: http://whatdowedoallday.com/dara-game (Accessed: 7 March 2018).

Garg, S., Songara, D. and Maheshwari, S. (2017) 'The winning strategy of Tic Tac Toe Game model by using Theoretical Computer Science', in *2017 International Conference on Computer, Communications and Electronics, (Comptelix)*. IEEE, pp. 89–95. doi:

10.1109/COMPTELIX.2017.8003944.
Horak, K., Bosanky, B. and Pechoucek, M. (2017) 'Heuristic Search Value Iteration for One-Sided Partially Observable Stochastic Games', in *Thirty-First AAAI Conference on Artificial Intelligence*, pp. 558–564.
Kabir, U. and AbdulHakim, I. (2019) 'Towards Search Algorithms for Combinatoiral Games', in *Proceedings of the 4th YUMSCIC*.
Kabir, U. and AbdulHakim, I. (2020) 'Evaluating Search Algorithms on Shiva Dara Game', *Dutse Journal of Pure and Applied Sciences*, 6(2), pp. 314–327.
Kim, S. K. (2017) 'A survey on issues at the GPGPU acceleration of the Monte Carlo tree search', *International Journal of Applied Engineering Research*, 12(15), pp. 4854–4857. Available at: http://www.ripublication.com.
Lelis, L. H. S. (2017) 'Stratified Strategy Selection for Unit Control in Real-Time Strategy Games', in *International Joint Conference on Artificial Intelligence (IJCAI)*, pp. 3735–3741.
Russell, S. J. and Norvig, P. (2009) *Artificial Intelligence: A Modern Approach*. Englewood Cliffs, New Jersey 07632: A Simon & Schuster Company, Prentice Hall, Inc.
Sato, M., Anada, K. and Tsutsumi, M. (2016) 'A Mathematical Formulation of the " Shicho " in the Game of Go', in *2016 IEEE/ACIS 15th International Conference on Computer and Information Science (ICIS)*. IEEE, pp. 1–6. doi: 10.1109/ICIS.2016.7550898.
Victor, do N. S. and Chaimowicz, L. (2017) 'MOBA: a New Arena for Game AI', *arxiv preprint arXiv:1705.10443*, pp. 1–8. Available at: http://arxiv.org/abs/1705.10443.
Yang, C. T. *et al.* (2017) 'Automatic tunable deployment for real-time strategy games', *Engineering Computations*, 34(2), pp. 239–250. doi: 10.1108/EC-08-2015-0251.
Zaslavsky, C. (1998) *Math Games and Activities from Around the World*. Chicago Review Press.
Zhang, Z. (2007) *Playing Tic-Tac-Toe Game Using Model Checking, CS 545 Course Project Spring*.