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A Mathematical Model on the Impact of Health Education Campaign on the Dynamics of Cigarette Smoking in a Varying Population

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Abstract
In this paper, a deterministic mathematical model on the dynamics of cigarette smoking in the presence of education-based intervention strategy has been developed and analyzed. Stability results revealed that the smoking-free equilibrium state is locally asymptotically stable under specified conditions. Eradication of smoking is achievable in finite time provided the smoking generation number, $R_0 < 1$. *Results of numerical experiments carried out on the model using MATLAB R2015a agree with the analytical results obtained. It is therefore concluded that health education campaign can play a significant role in controlling the dynamics of cigarette smoking.*

Keywords: Cigarette, mathematical model, education-based intervention, generation number, locally asymptotically stable.

INTRODUCTION
Smoking is a practice in which a substance, in this case, the leaves of tobacco plant, is burnt and the resulting smoke is then inhaled and the active substances absorbed through the lungs into the bloodstream. Cigarette smoking is the most popular form of smoking being practiced by over one billion people globally, which majority of whom live in developing countries. Nearly eighty percent of the over one billion smokers worldwide are found in low-and middle-income countries, where the burden of cigarette-related illness and death is heaviest WHO (2016).

Diseases related to cigarette smoking have been shown to kill approximately half of long-term smokers when compared to average mortality rates faced by non-smokers. The World Health Organization estimated that cigarette causes approximately 6 million deaths annually worldwide and this number is expected to rise to 8 million by the year 2030. It is obvious that the risk of death is highly increasing with the high rate of cigarette consumption. Cigarette has become a major cause for many killer diseases CDC (2016).

Castilo, Jordan and Rodriguez (2000) presented a simple mathematical model for giving up smoking. They considered a system with a total constant population which is divided into three classes: potential smokers, that is, people who do not smoke yet but might become

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smokers in the future (P), smokers (S), and people (former smokers) who have quit smoking permanently (Q). Later, Sharomi and Gumel (2008) introduced a new class Q_t of smokers who temporarily quit smoking. They concluded that the smoking-freeequilibrium state is globally-asymptotically stable whenever the threshold is less than unity, otherwise it is unstable. The public health implication of this result was that the number of smokers can be effectively controlled (or eliminated) if the threshold is made to be less than unity. Zaman (2011) derived and analyzed a mathematical model on smoking taking into account the occasional smokers compartment. He also extended the model to consider the possibility of quitters becoming smokers again and presented its qualitative behavior.

Robert and Joshua (1999) examined how smoking behavior spreads within specific subgroups of the population based on the density of network interactions both within and across subgroups. These include the tendency for individuals to follow social norms: either passively, through (peer imitation) or actively, in response to encouragement (peer pressure). Lock (2010) analyzed smokers' behavior in different ethnic groups using a longitudinal and qualitative study of smokers. As proposed, tobacco smoking can be considered as a socially transmitted habit, Christakis and Fowler (2009).

Most people start smoking when they are in their teen age and become addicted by the time they become adults. Since teens see older people around them smoking especially their parents, relatives, friends or peer, they may feel pressured into doing the same. It is estimated that the probability of a son or daughter smoking if both parents smoke is 24% or 23% but this falls to almost 12% if neither of the parent smokes Loureiro, Galdeano and Vuri (2012). While, adults smoke when they have a lot of pressure and stress because of the economic or personal problems, some have tried to quit but have returned to smoking because of addiction Jacobs (1997).

Cigarette smoking is the leading cause of preventable deaths worldwide. Treatment of smoking-related illness cost billions of naira each year, imposing a heavy economic toll on countries both in terms of direct medical care and media campaigns WHO (2016). It poses enormous health- and non-health-related costs to the affected individuals, employers and the society at large Epku and Brown (2015). Research shows that an average smoker in Nigeria spends an average sum of N1,202.5 on cigarette per month. On the whole, Nigerian smokers spend an average of N7.45 billion on smoking monthly, and N89.5 billion yearly Shittu (2015).

Zaman (2009) investigated the qualitative behavior of giving up smoking model in a varying population accounting for occasional smokers compartment with bilinear incidence rate. Analysis of the model revealed that there were four equilibrium states including thesmoking-free equilibrium state. Osu and Olunkwa (2010) considered the introduction of new brands of cigarette into the market and (or) the introduction of new smokers in a given population. The model was used to predict the smoke attributed mortality over a period of 20 years, using empirical data of Nigerian population. They observed that increasing the prices of tobacco products was arguably the most effective method of cutting the prevalence of tobacco use and reducing consumption of tobacco products.

Lahrouz, Omari, Kiouach and Belmaâti (2011) constructed a Lyapunov function to prove the global stability of the unique smoking-endemic equilibrium state of a mathematical model, the solution to the mathematical model was obtained by a stochastic Lyapunov method. Their study revealed that the deterministic model has a globally asymptotically stable smoking-free equilibrium, if the threshold quantity was less than 1. The smoking-endemic equilibrium state was also found to be globally asymptotically stable.

Levy, Graham, Mabry, Abrams and Orleans (2010) modeled the impact of smoking-cessation treatment policies on quit rates. Simulation results demonstrated that cessation treatment policies could have a major impact on smoking rates, reducing smoking prevalence from its current rate of 20.5% to 17.2% in the first year with continued reductions in future years. For policies to be optimally effective, they must support a comprehensive, seamless system of care management at every level of societal structure. The stability analyses of mathematical models on giving up smoking, effect of heavy smokers on the dynamics of a smoking, and the effect of occasional smokers on the dynamics of a smoking were carried by Alkhudhari, Al-Sheik and Al-Tuwairqi (2015), Alkhudhari, Al-Sheik and Al-Tuwairqi (2014a),Alkhudhari, Al-Sheik and Al-Tuwairqi (2014b).

The effect of cigarette smoking brings big problems both in personal and public matters, especially at a time where many of our youths embrace smoking. Clearly cigarette smoking is a prevalent problem that requires intervention for control and possibly eradication. Therefore, mathematical models are useful tools in understanding the dynamics of smoking for better control and eradication.

MODEL FORMULATION

Model Description

We consider a population divided into seven compartments namely; Uneducated potential smokers, P_u(t); Educated potential smokers, P_E(t); Non-smokers, $P_N(t)$; Occasional smokers, $O(t)$; Smokers, $S(t)$; Smokers who temporarily quit smoking, $Q_T(t)$, and Smokers who permanently quit smoking, $Q_P(t)$, all at time t. The movement of individuals from one compartment to another is as described in figure 1. We assume a recruitment into the non-smoking compartment occurs at a rate Λ. It is also assumed that individuals in all compartments die due to causes other than cigarette smoking at a rate μ_1, while smoking is an additional cause of death among smokers at a rate μ_2. The parameters $\kappa_1, \kappa_2, \delta, \alpha, \gamma$ and σ are the rate at which uneducated potential smokers get educated, rate at which the educated potential smokers move to non-smokers compartment, rate at which educated potential smokers who disregard their knowledge of the dangers of smoking proceed to become occasional smokers, contact rate between uneducated potential smokers and occasional smokers, contact rate between occasional smokers and smokers, rate at which temporary quitters revert back to smoking, and the rate at which smokers quit smoking, respectively.

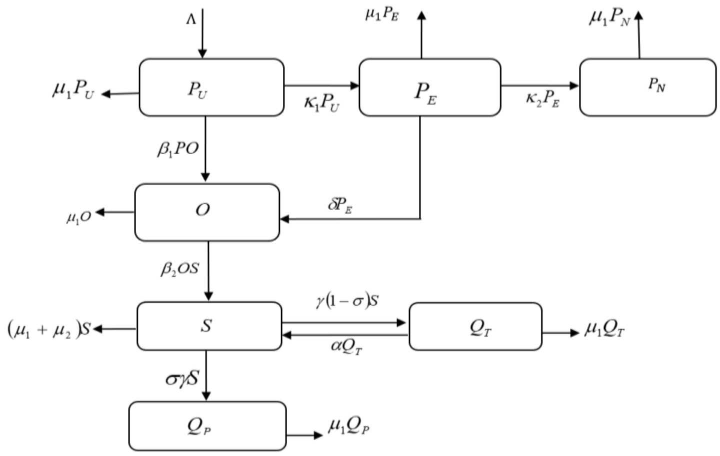

Figure 1: Model flow diagram

The Model Equations

The model equations formulated from a combination of the model flow diagram (see fig.1) and the model description provided under section 2.1 are presented in equations (1) to (7).

$$\frac{dP_U}{dt} = \Lambda - (\beta_1 O + \kappa_1 + \mu_1)P_U \quad (1)$$

$$\frac{dP_E}{dt} = \kappa_1 P_U - (\kappa_2 + \delta + \mu_1)P_E \; ; \quad (2)$$

$$\frac{dP_N}{dt} = \kappa_2 P_E - \mu_1 P_N \quad (3)$$

$$\frac{dO}{dt} = (\beta_1 P_U - \beta_2 S - \mu_1)O + \delta P_E \quad (4)$$

$$\frac{dS}{dt} = \beta_2 OS + \alpha Q_T - (\mu_1 + \mu_2 + \gamma)S \quad (5)$$

$$\frac{dQ_T}{dt} = \gamma(1-\sigma)S - (\mu_1 + \alpha)Q_T \quad (6)$$

$$\frac{dQ_P}{dt} = \sigma\gamma S - \mu_1 Q_P \quad (7)$$

With initial conditions $P_U(0) = P_{U0}$, $P_E(0) = P_{E0}$, $P_N(0) = P_{N0}$, $O(0) = O_0$, $S(0) = S_0$, $Q_T(0) = Q_{T0}$, $Q_P(0) = Q_{P0}$ (8)

$$T(t) = P_U(t) + P_E(t) + P_N(t) + O(t) + s(t) + Q_T(t) + Q_P(t) \quad (9)$$

where, $T(t)$ is the total population at time t.

RESULTS AND DISCUSSION

Existence of invariant region

To investigate the existence of an invariant region, we differentiate equation (9) and apply equations (1) to (7) to obtain

$$\frac{dT}{dt} = \Lambda - \mu_1 T - \mu_2 \text{S}, T(0) = T_0 \quad (10)$$

$$\frac{dT}{dt} \leq \Lambda - \mu_1 T \quad (11)$$

Integrating equation (11) and applying the initial condition of (10), we have

$$T(t) \leq \frac{\Lambda}{\mu_1} + \left(T_0 - \frac{\Lambda}{\mu_1}\right)e^{-\mu_1 t} \quad (12)$$

Taking of equation (12) as $t \to \infty$, we have

$$\operatorname*{Lim}_{t\to\infty} T(t) \leq \lim_{t\to\infty} \frac{\Lambda}{\mu_1} + \left(T_0 - \frac{\Lambda}{\mu_1}\right)e^{-\mu_1 t}$$

Clearly,

$$\operatorname*{Lim}_{t\to\infty} T(t) \leq \frac{\Lambda}{\mu_1}$$

since, $\lim_{t\to\infty} e^{-\mu_1 t} = 0$

Therefore, the region in which the mathematical model is biological meaningful is given by

$$\Omega = \left\{(P_U, P_E, P_N, O, S, Q_T, Q_P) \in \Re_+^7 : P_U + P_E + P_N + O(t) + S + Q_T + Q_P \leq \frac{\Lambda}{\mu_1}\right\} \quad (13)$$

The implication of the region (13) is that every solution starting withinΩremains in Ω for all$t >$ 0. Therefore, in the regionΩ, the mathematical model under review feasible, well-posed and positively invariant.

Existence of smoking-free equilibrium State

Smoking-free equilibrium,E_0, is a state at which smoking has been completely eradicated from the entire population. Substituting $O(t) = S(t) = Q_T(t) = Q_P(t) = 0$ into equations (1) to (7), equate the right hand sides of the same set of equations to zero and solving the resulting system of equations simultaneously we obtain E_0 as given by equation (14).

$$E_0 = ((P_U, P_E, P_N, O, S, Q_T, Q_P) = \left(\frac{\Lambda}{\kappa_1+\mu_1}, \frac{\Lambda}{(\kappa_1+\mu_1)}, \frac{\Lambda}{\mu_1(\kappa_1+\mu_1)(\kappa_1+\delta+\mu_1)}, 0,0,0,0\right) \quad (14)$$

Smoking generation number

The smoking generation number denoted by,R_0, can be defined as the number of new smokers produced by a typical smoker in a population at a smoking free equilibrium. It is an important threshold parameter that determines whether or not; smoking will spread through a given population.

We apply the next generation matrix technique by Diekmann, Heesterbeek and Metz (1990) and Van den Driessche and Watmough (2002) to obtain the smoking generation number, R_0.

Let $U_i$be the rate of emergence of new smokers in the i compartment and V_i be the rate of transfer of individuals out of the$i$compartment, given the smoking-free equilibrium.$R_0$is the spectral radius (largest eigenvalue) of the next generation matrix denoted byUV^{-1}.

Let $X = (O, S, Q_T)^T$,

which can be written in the form

$$\frac{dx}{dt} = U_i(x) - V_i(x) \quad (15)$$

Where

$$U_i(x) = \begin{pmatrix} U_1 \\ U_2 \\ U_3 \end{pmatrix} = \begin{pmatrix} \beta_1 P_U O \\ \beta_2 OS \\ O \end{pmatrix} \quad (16)$$

and

$$V_i(x) = \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = \begin{pmatrix} \beta_2 OS + \delta P_E \\ (\mu_1 + \mu_2 + \gamma)S - \alpha Q_T \\ (\mu_1 + \alpha)Q_T - \gamma(1-\sigma)S \end{pmatrix} \quad (17)$$

The Jacobian matrix is given by

$$J = \begin{pmatrix} \frac{\partial U_1}{\partial O} & \frac{\partial U_1}{\partial S} & \frac{\partial U_1}{\partial Q_T} \\ \frac{\partial U_2}{\partial O} & \frac{\partial U_2}{\partial S} & \frac{\partial U_2}{\partial Q_T} \\ \frac{\partial U_3}{\partial Q_T} & \frac{\partial U_3}{\partial Q_T} & \frac{\partial U_3}{\partial Q_T} \end{pmatrix} \tag{18}$$

Evaluating the Jacobian matrix (21) at E_0, we obtain

$$J_{(E_0)} = \begin{pmatrix} \frac{\Lambda}{\kappa_1+\mu_1} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{19}$$

Similarly, the Jacobian matrix for $V_i(x)$

$$J = \begin{pmatrix} \frac{\partial V_1}{\partial O} & \frac{\partial V_1}{\partial S} & \frac{\partial V_1}{\partial Q_T} \\ \frac{\partial V_2}{\partial O} & \frac{\partial V_2}{\partial S} & \frac{\partial V_2}{\partial Q_T} \\ \frac{\partial V_3}{\partial Q_T} & \frac{\partial V_3}{\partial Q_T} & \frac{\partial V_3}{\partial Q_T} \end{pmatrix} \tag{20}$$

Evaluating the Jacobian (23) at E_0, we obtain

$$J_{(E_0)} = \begin{pmatrix} \mu_1 & 0 & 0 \\ 0 & \mu_1+\mu_2+\gamma & -\alpha \\ 0 & -\gamma(1-\sigma) & \mu_1+\alpha \end{pmatrix} \tag{21}$$

$$V^{-1} = \begin{pmatrix} \frac{1}{\mu_1} & 0 & 0 \\ 0 & \frac{1}{(\mu_1+\mu_2+\gamma)-\alpha\gamma(1-\sigma)} & \frac{-\alpha}{(\mu_1+\mu_2+\gamma)(\mu_1+\alpha)-\alpha\gamma(1-\sigma)} \\ 0 & \frac{-\gamma(1-\sigma)}{(\mu_1+\mu_2+\gamma)(\mu_1+\alpha)-\alpha\gamma(1-\sigma)} & \frac{\mu_1+\alpha}{(\mu_1+\mu_2+\gamma)-\alpha\gamma(1-\sigma)} \end{pmatrix} \tag{22}$$

The characteristic polynomial is given by

$$|UV^{-1} - \lambda I| = 0 \tag{23}$$

Using equations (21) and (22) in (23), we obtain

$$\begin{vmatrix} \frac{\Lambda\beta_1}{\mu_1(\kappa_1+\mu_1)} - \lambda & 0 & 0 \\ 0 & -\lambda & 0 \\ 0 & 0 & -\lambda \end{vmatrix} = 0$$

or $$\lambda^2\left(\frac{\Lambda\beta_1}{\mu_1(\kappa_1+\mu_1)} - \lambda\right) = 0 \tag{24}$$

The eigenvalues are therefore the solutions to equation (24)

$$\lambda_1 = \lambda_2 = 0;\ \lambda_3 = \frac{\Lambda\beta_1}{\mu_1(\kappa_1+\mu_1)}$$

By definition $R_0 = max\{|\lambda_i|: \lambda_i\ are\ the\ eigenvalues\ of\ UV^{-1}\}$

Hence, the smoking generation number, R_0 is obtained as

$$R_0 = \frac{\Lambda\beta_1}{\mu_1(\kappa_1+\mu_1)} \tag{25}$$

Local Stability of the smoking free equilibrium

Theorem 1

The smoking-free equilibrium point,E_0, of the mathematical model under review is given by equations (1)– (7) is locally asymptotically stable if$R_0 < 1$ and unstable if $R_0 > 1$.

Proof:

Let the RHS of equations (1) to (7) be equal to $F_1, F_2, F_3, F_4, F_5, F_6$ and F_7, respectively.

The Jacobian matrix of the resulting system of equations is given by

$$J = \begin{pmatrix} \frac{\partial F_1}{\partial P_U} & \frac{\partial F_1}{\partial P_E} & \frac{\partial F_1}{\partial P_N} & \frac{\partial F_1}{\partial O} & \frac{\partial F_1}{\partial S} & \frac{\partial F_1}{\partial Q_T} & \frac{\partial F_1}{\partial Q_P} \\ \frac{\partial F_2}{\partial P_U} & \frac{\partial F_2}{\partial P_E} & \frac{\partial F_2}{\partial P_N} & \frac{\partial F_2}{\partial O} & \frac{\partial F_2}{\partial S} & \frac{\partial F_2}{\partial Q_T} & \frac{\partial F_2}{\partial Q_P} \\ \frac{\partial F_3}{\partial P_U} & \frac{\partial F_3}{\partial P_E} & \frac{\partial F_3}{\partial P_N} & \frac{\partial F_3}{\partial O} & \frac{\partial F_3}{\partial S} & \frac{\partial F_3}{\partial Q_T} & \frac{\partial F_3}{\partial Q_P} \\ \frac{\partial F_4}{\partial P_U} & \frac{\partial F_4}{\partial P_E} & \frac{\partial F_4}{\partial P_N} & \frac{\partial F_4}{\partial O} & \frac{\partial F_4}{\partial S} & \frac{\partial F_4}{\partial Q_T} & \frac{\partial F_4}{\partial Q_P} \\ \frac{\partial F_5}{\partial P_U} & \frac{\partial F_5}{\partial P_E} & \frac{\partial F_5}{\partial P_N} & \frac{\partial F_5}{\partial O} & \frac{\partial F_5}{\partial S} & \frac{\partial F_5}{\partial Q_T} & \frac{\partial F_5}{\partial Q_P} \\ \frac{\partial F_6}{\partial P_U} & \frac{\partial F_6}{\partial P_E} & \frac{\partial F_6}{\partial P_N} & \frac{\partial F_6}{\partial O} & \frac{\partial F_6}{\partial S} & \frac{\partial F_6}{\partial Q_T} & \frac{\partial F_6}{\partial Q_P} \\ \frac{\partial F_7}{\partial P_U} & \frac{\partial F_7}{\partial P_E} & \frac{\partial F_7}{\partial P_N} & \frac{\partial F_7}{\partial O} & \frac{\partial F_7}{\partial S} & \frac{\partial F_7}{\partial Q_T} & \frac{\partial F_7}{\partial Q_P} \end{pmatrix} \tag{26}$$

Evaluating equation (29) at the smoking-free equilibrium state, E_0 in equation (17),we obtain (30).

$$J(E_0) = \begin{pmatrix} -(\kappa_1+\mu_1) & 0 & 0 & -\frac{\beta_1\Lambda}{\kappa_1+\mu_2} & 0 & 0 & 0 \\ \lambda_1 & -(\kappa_2+\delta+\mu_1) & 0 & 0 & 0 & 0 & 0 \\ 0 & \kappa_2 & -\mu & 0 & 0 & 0 & 0 \\ 0 & \delta & 0 & \frac{\beta_1\Lambda}{\kappa_1+\mu_1}-\mu_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -(\mu_1+\mu_2+\gamma) & \alpha & 0 \\ 0 & 0 & 0 & 0 & \gamma(1-\sigma) & -(\mu_1+\alpha) & 0 \\ 0 & 0 & 0 & 0 & \sigma\gamma & 0 & -\mu \end{pmatrix} \tag{27}$$

Next, we used elementary row-operations to reduce the Jacobian matrix evaluated at smoking free equilibrium as used by Abdulrahman, Akinwande, Awojoyogbe and Abubakar (2013).

$C_1 = \beta_1\kappa_1$, $C_3 = \gamma(1-\sigma)$, $C_4 = \sigma\gamma$ where $\kappa_1 = \frac{\Lambda}{\kappa_1+\mu_1}$

$D_1 = (\kappa_1+\mu_1)$, $D_2 = (\kappa_2+\delta+\mu_1)$, $D_3 = (\mu_1+\mu_2+\gamma)$, $D_4 = (\mu_1+\alpha)$

Now transforming (30) into upper triangular matrix using elementary row-transformation, we obtain

$$\left|J\left(E_0\right)-\lambda I\right|=\begin{pmatrix} -D_1-\lambda & 0 & 0 & -C_1 & 0 & 0 & 0 \\ 0 & -D_2-\lambda & 0 & -M_1 & 0 & 0 & 0 \\ 0 & 0 & -\mu_1-\lambda & -M_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -M_3-\mu_1-\lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -D_3-\lambda & \alpha & 0 \\ 0 & 0 & 0 & 0 & 0 & M_4-\lambda & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\mu-\lambda \end{pmatrix}=0 \quad (28)$$

Evaluating the determinant of (28) yield these eigenvalues:

$$\lambda_1 = -D_1 = -(\kappa_1 + \mu_1)$$

$$\lambda_2 = -D_2 = -(\kappa_2 + \delta + \mu_1)$$

$$\lambda_3 = -\mu_1$$

$$\lambda_4 = -M_3 = -\frac{C_1\kappa_1\delta}{D_1D_2} + C_1 - \mu_1 = -\frac{\beta_1\Lambda\kappa_1\delta}{(\kappa_1+\mu_1)^2(\kappa_1+\delta+\mu_1)} + \frac{\beta_1\Lambda}{\kappa_1+\mu_1} - \mu_1$$

$$\lambda_5 = -D_3 = -(\mu_1 + \mu_1 + \gamma)$$

$$\lambda_6 = M_4 = \frac{\alpha C_3}{D_3} - D_4 = \frac{\alpha C_3 - D_3D_4}{D_3} = \frac{\alpha\gamma(1-\sigma)-(\mu_1+\mu_2+\gamma)(\mu_1+\alpha)}{(\mu_1+\mu_2+\gamma)}$$
$$= \frac{-[(\mu_1+\mu_2)(\mu_1+\alpha)+\gamma(\mu_1+\alpha\sigma)]}{(\mu_1+\mu_2+\gamma)}$$

$$\lambda_7 = -\mu_1$$

$$\lambda_4 = \frac{-\beta_1\Lambda\kappa_1\delta}{\mu_1(\kappa_1-\mu_1)^2(\kappa_2+\delta+\mu_1)} + \frac{\beta_1\Lambda}{\mu_1(\kappa_1+\mu_1)} - 1$$

But

$$R_0 = \frac{\beta_1\Lambda}{\mu_1(\kappa_1+\mu_1)}$$

$$\Rightarrow \lambda_4 = -\frac{\beta_1\Lambda\kappa_1\delta}{\mu_1(\kappa_1+\mu_1)^2(\kappa_2+\delta+\mu_1)} + R_0 - 1$$

Since parameters are positive

Let $\theta = \frac{\beta_1\Lambda\kappa_1\delta}{\mu_1(\kappa_1+\mu_1)^2(\kappa_2+\delta+\mu_1)}$ this implies $\theta > 0$. Then we have $\lambda_4 = -\theta + (R_0 - 1)$

Thus $\lambda_4$is only negative if $R_0 < 1$ satisfies the negativity requirement for stability.

Therefore, all the eigenvalues $\lambda_i < 0$for $i = 1, 2, 3, 4, 5, 5\ 6,7$ when $R_0 < 1$ we conclude that the smoking-free equilibrium point is locally asymptotically stable.

Numerical Experiments

We performed some numerical experiments using ode45 function from MATLAB R2010a to study the effect of health education campaign on the dynamicsthe system of model equations. The parameter values for the simulations are provided in table 2.

Table 2: Variables and parameters values used for computational results.

Variables	Values	References
Λ	1, 0.15	Alkhudari et al., (2014)
β_1	0.04	Alkhudari et al., (2015)
β_2	0.3	Alkhudari et al., (2015)
γ	0.0 - 0.2	Alkhudari et al., (2014)
σ	0.05 - 0.15	Alkhudari et al., (2014)
α	0.25	Alkhudari et al., (2014)
μ_1	0.15	Alkhudari et al., (2014)
μ_2	0.30	HSCIC., (2016)
κ_1	0.0 - 0.75	Assumed
κ_2	0.0 - 0.50	Assumed
δ	0.0-0.23	Assumed
$P_U(0)$	0.520	Assumed
$P_E(0)$	0.085	Assumed
$P_N(0)$	0.040	Assumed
$O(0)$	0.210	Ullah, et al., (2016)
$S(0)$	0.095	Ullah, et al., (2016)
$Q_T(0)$	0.035	Ullah, et al., (2016)
$Q_P(0)$	0.015	Ullah, et al., (2016)

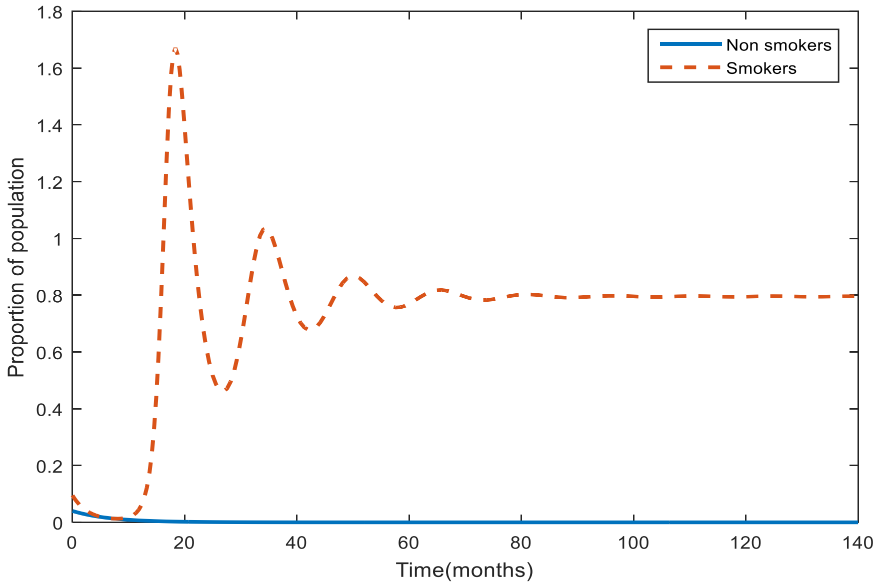

Figure 2: The behavior of the model in the absence of control measures. Parameter values used are as in Table 2 with $\kappa_1 = \kappa_2 = \delta = 0, \gamma = 0.05, \sigma = 0.10$

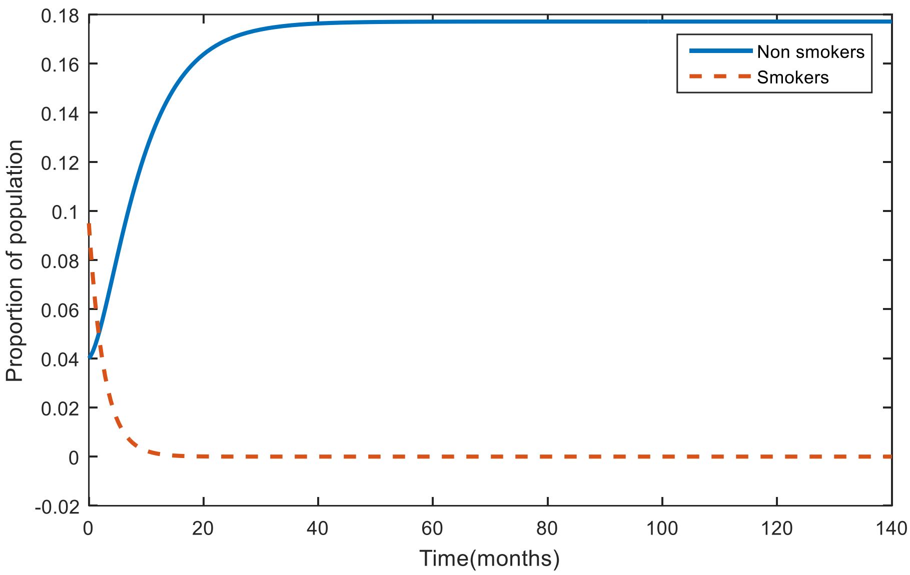

Figure 3: The behavior of the model when$\kappa_1 = 0.15, \kappa_2 = 0.10, \delta = 0.03, \gamma = 0.05,$

$\sigma = 0.10$

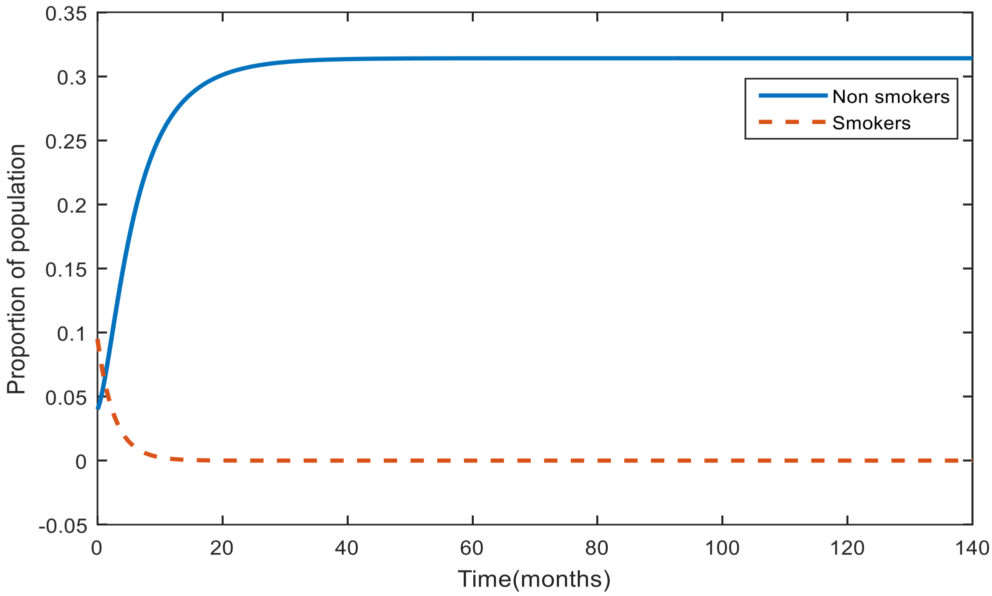

Figure 4: The behavior of the model when$\kappa_1 = 0.30, \kappa_2 = 0.20, \delta = 0.07, \gamma = 0.05, \sigma = 0.10$

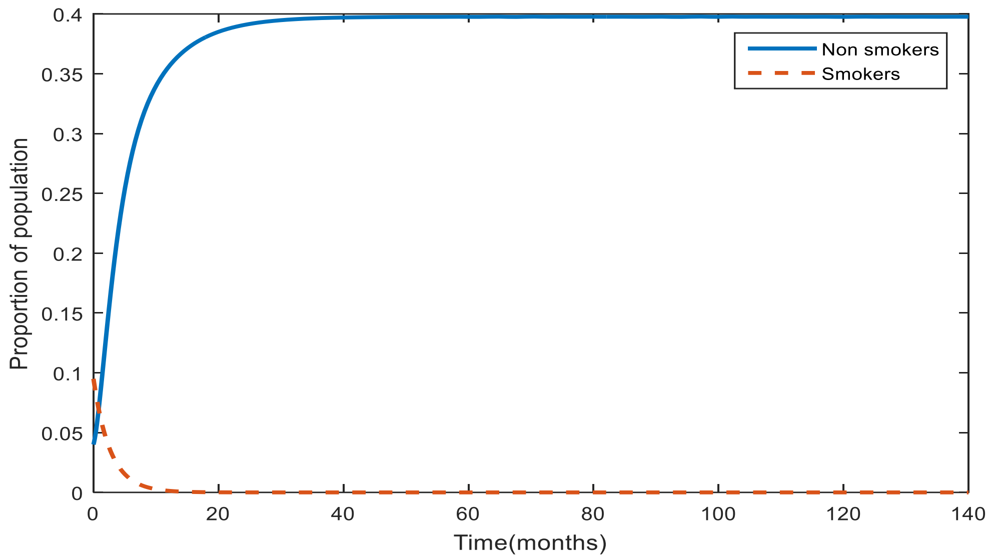

Figure 5: The behavior of the model when$\kappa_1 = 0.45, \kappa_2 = 0.30, \delta = 0.11, \gamma = 0.05,$

$\sigma = 0.10$

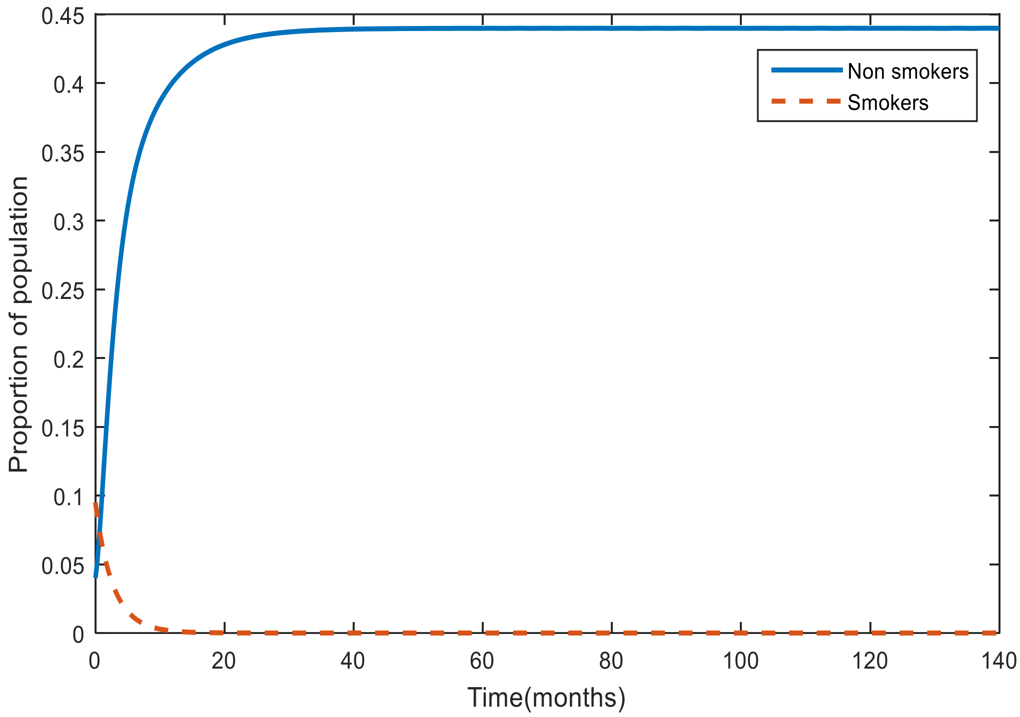

Figure 6: The behavior of the model when$\kappa_1 = 0.60, \kappa_2 = 0.40, \delta = 0.17, \gamma = 0.05, \sigma = 0.10$

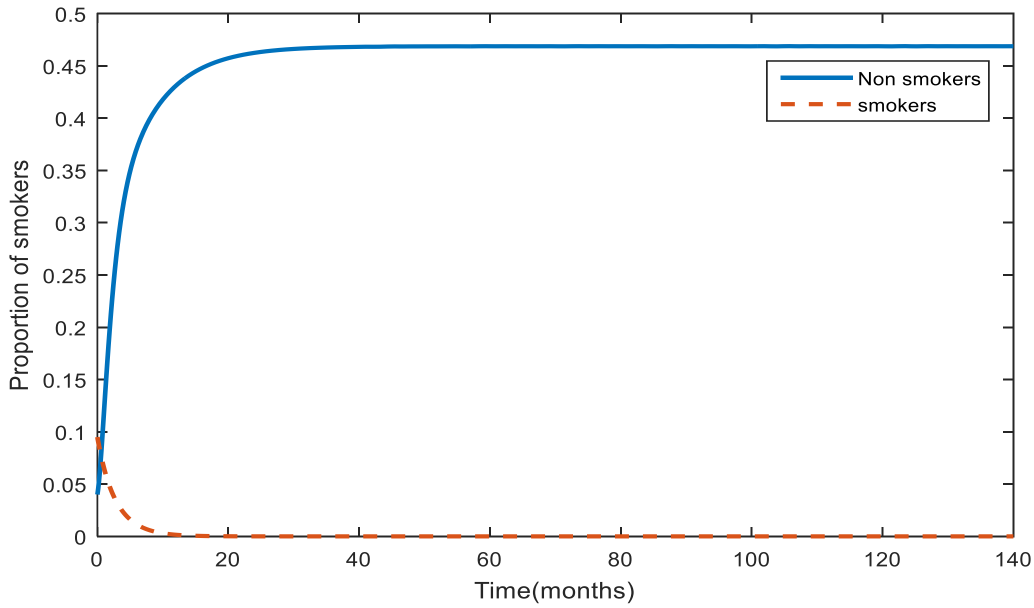

Figure 7: The behavior of the model when$\kappa_1 = 0.75, \kappa_2 = 0.50, \delta = 0.23, \gamma = 0.05, \sigma = 0.10$

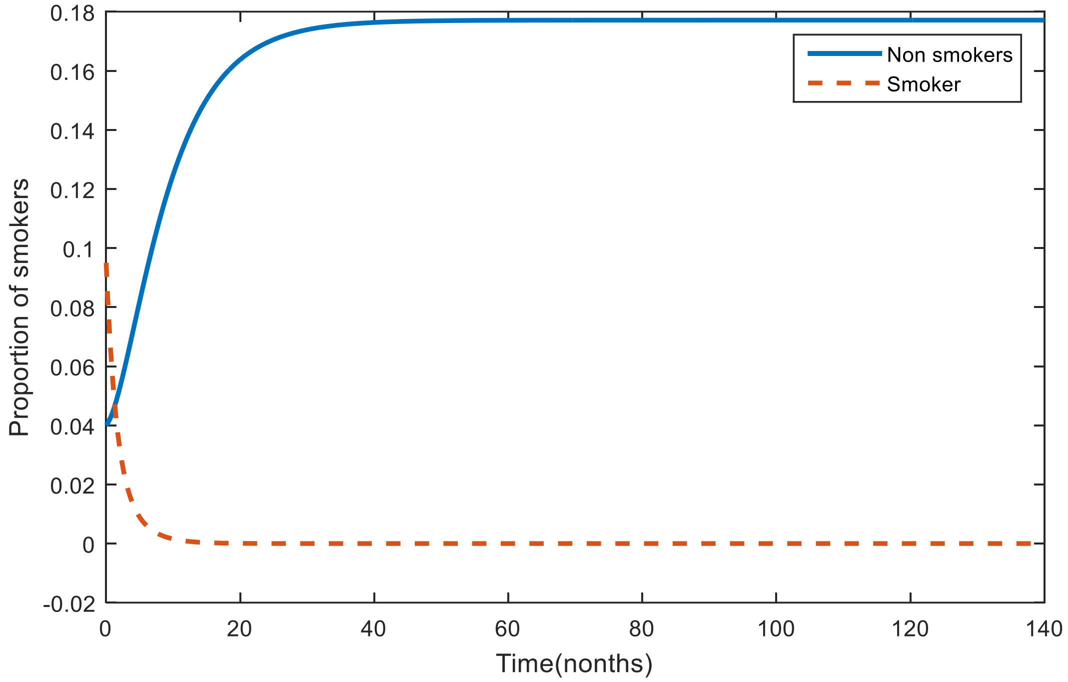

Figure 8: The behavior of the model when$\kappa_1 = 0.15, \kappa_2 = 0.10, \delta = 0.03, \gamma = 0.10,$ $\sigma = 0.15$

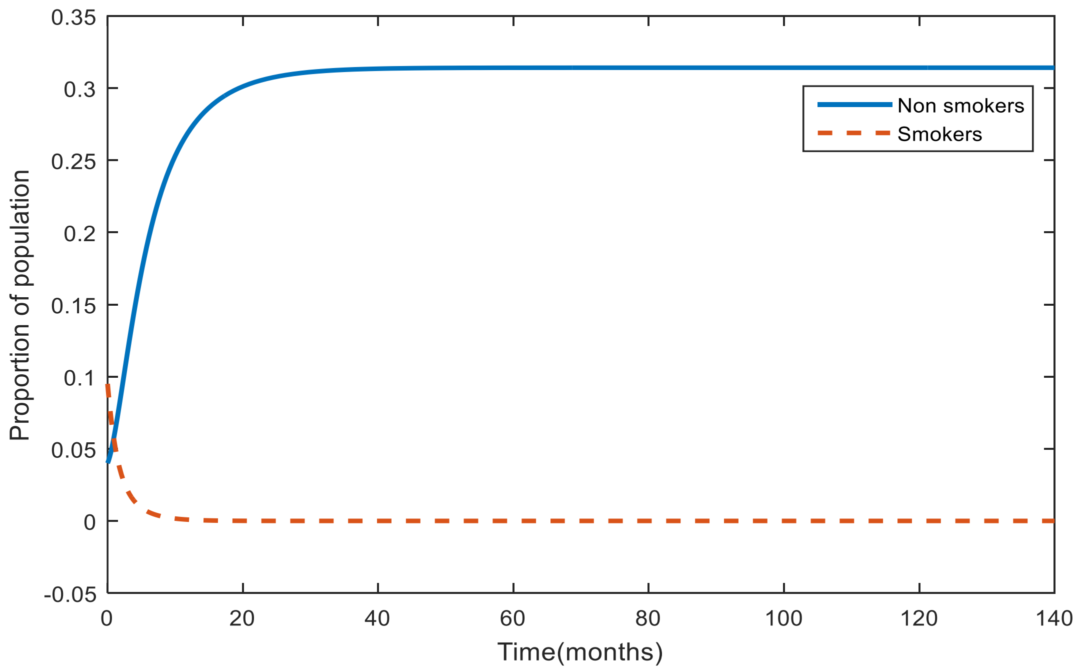

Figure 9: The behavior of the model when$\kappa_1 = 0.30, \kappa_2 = 0.20, \delta = 0.07, \gamma = 0.10, \sigma = 0.15$

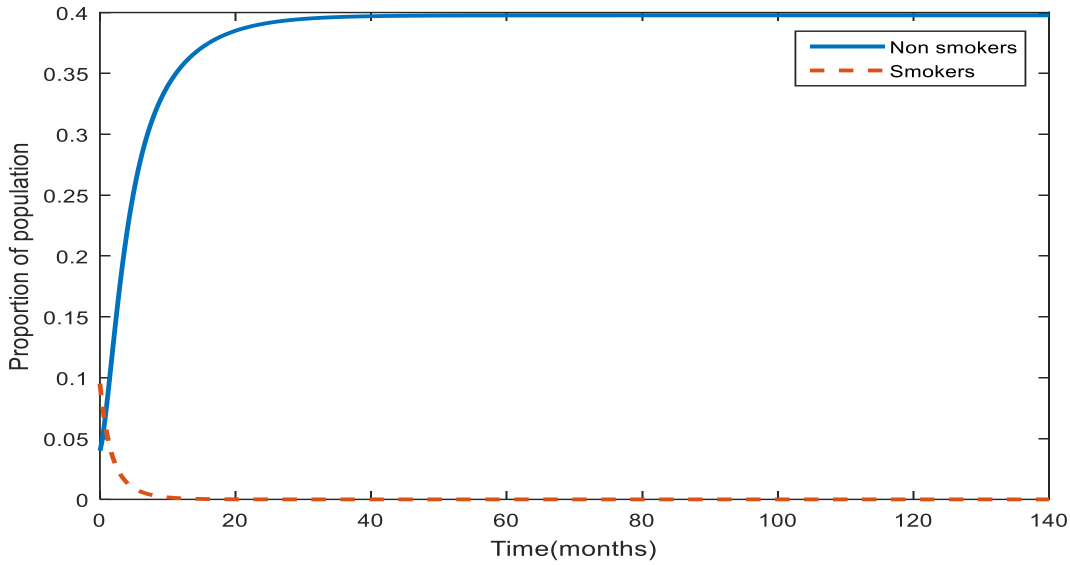

Figure 10: The behavior of the model when$\kappa_1 = 0.45, \kappa_2 = 030, \delta = 0.11, \gamma = 0.10, \sigma = 0.15$

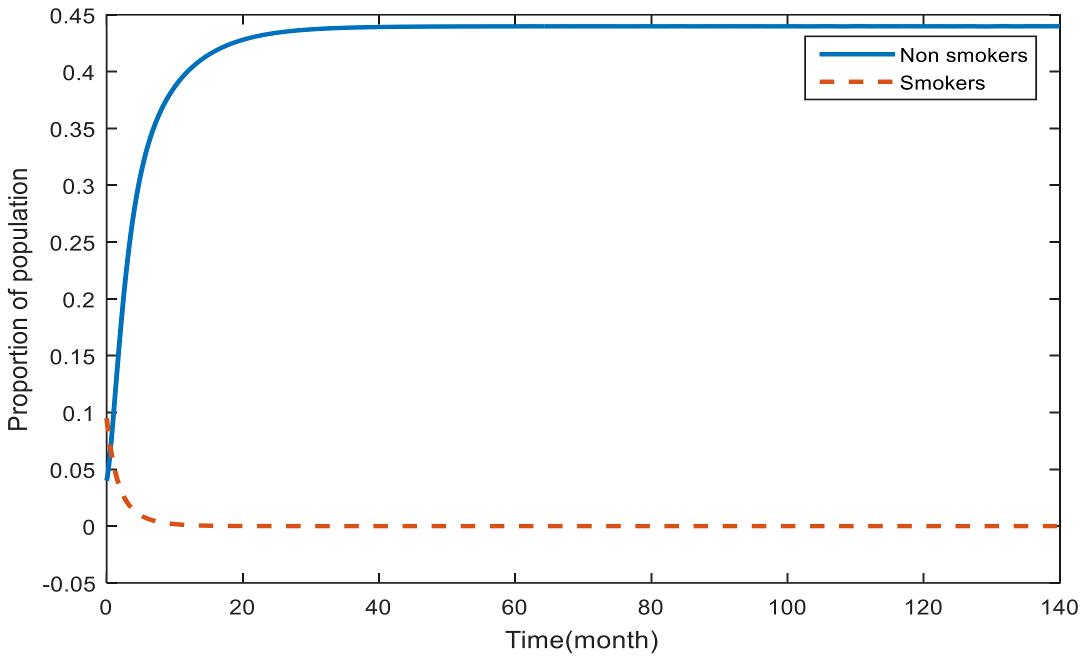

Figure 11: The behavior of the model when$\kappa_1 = 0.60, \kappa_2 = 0.40, \delta = 0.17, \gamma = 0.10, \sigma = 0.15$

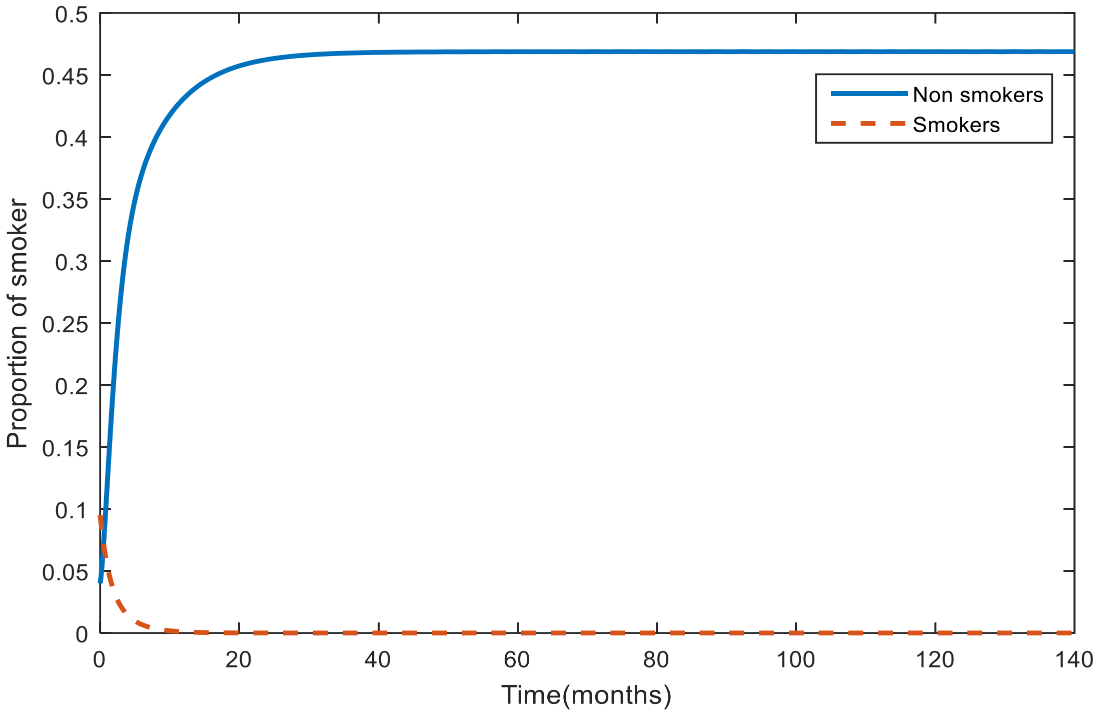

Figure 12: The behavior of the model when$\kappa_1 = 0.75, \kappa_2 = 0.50, \delta = 0.23, \gamma = 0.10,$
$\sigma = 0.15$

DISCUSSION OF RESULTS

We established the existence of invariant region of the system of model equations denoted by Ω given by equation (13). The smoking free equilibrium denoted by E_0 was established for the system (1) to (7) and given by equation (14). We then obtained the smoking generation numberR_0, using next generation matrix which is given by equation (25). We further established the local stability of smoking-free equilibriumof the system of model equations. Stability results show that all the eigenvalues of the row–transformed Jacobian matrix evaluated at the smoking-free equilibrium are real and negative. This implies that the model under review is locally asymptotically stable if $R_0 < 1$.This is physical interpreted to mean that smoking can be eradicated in the population in finite time under the specified conditions.

In the absence of health education campaign, the proportion of smokers oscillates until it stabilizes at a very high value of 0.8 while that of non-smokers decreases and approaches zero (see figure 2).

Maintaining the rates of quitting smoking by temporary and permanent quitters at 0.05 and 0.10,respectively while varying the rate of health education campaign to 15%, 30%, 45%, 60% and 75% as control measures,we observe decrease in the proportion of smokers, while that of non-smokers continue increase (see figures (3), (4), (5), (6) and (7). This numerical resultsagree with analytical results obtained in this research that increasing health education campaign leads to rapid increase in the proportion of non-smokers implying that smoking could be eradicated in finite time.

In figure (8) - (12), the rate of quitting smoking by temporary and permanent quitters was increased by 0.10 and 0.15 respectively while the rate of health education campaign was varied at 15%, 30%, 45%, 60% and 75%. These results show rapid increase in the proportions of non-smokers and also rapid decrease in the proportions of smokers as a result of increased in the rate of health education campaign.

CONCLUSION

A mathematical model for the dynamics of smoking in a varying population considering health education campaign as an intervention strategy has been developed and studied. The invariant region has been obtained and the domain of the system is physically feasible. The model developed has been analytically studied using the row–transformation technique. The smoking-free equilibrium point of the system of model equations is found to be locally asymptotically stable if $R_0 < 1$. The results of the numerical experiments carried out agree with the analytical findings that health education campaign could significantly reduce the proportion of smokers in the population.

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