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Conventional Implicit Uniform Order Second Derivative Block Backward Differentiation Formula for Numerical Solution of Stiff Ordinary Differential Equations

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Abstract

Second derivative conventional block backward differentiation formula method of order 6 was constructed for the numerical solution of stiff initial value problems. Each scheme was obtained through increasing the number k in the multi-step collocation (MC) with the aid of maple 18 software. In addition, the order and error constant have been investigated, we also compared the newly derived method with exact solutions and have shown that the results obtained using proposed block methods are excellent for the solution of stiff problems.

Keywords: Conventional, Block Method, Backward Differentiation Formula, Second Derivative, Stiff ODEs

INTRODUCTION

Differential equations play a role in the modeling of almost every scientific discipline. However, it is relatively rare for a differential equation to have a solution that can be written in terms of elementary functions Butcher (1997). Differential equation can describe merely all system undergoing change. In some cases, the differential equations could be solved analytically while others may be too complicated to use analytic methods of solutions. Numerical methods are often used to give approximate solutions to these differential equations. Olabode (2013). Oluwaseun *et al* (2018) proposed block multistep method for the direct Solution of third order of ordinary differential equations. The method of collocation and interpolation of power series approximate solution was used to derive the continuous linear multistep method. The third derivative block method is adopted to solve some special third order numerical problems previously solved in literature and the method gave better results. The convergence, order, error constant and zero-stability of the new methods were investigated. The performance of the new block method was tested with some third order initial value problems. Chollom *et al* (2004) and Chollom (2005). These methods were used in providing sufficient number of finite difference equations simultaneously in block form for the numerical solutions of first order ordinary differential equations. Adesanya *et al* (2008) developed an improved Continuous Linear Multistep Method for direct solution of the general second order ordinary differential equations. The approach is based on the collocation of the differential system and interpolation of the approximate solution. The efficiency of this scheme was tested with some numerical examples. In this research work

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continuous conventional second derivative block backward differentiation formula linear multistep methods with order 4 and error constants are developed for implementation as proposed by Yakubu et al (2014)

Derivation of Method

The derivation techniques of two step conventional block method for solution of stiff systems of initial value problems. In this regard, the derivation will be carried out through the interpolation and collocation on the equi-distance step points.

Let the approximate solution be given as power series of a single variable x in the form

$$y_k(x) = \sum_{j=0}^{m-1} \alpha_j x^j \tag{1}$$

$$y'_k(x) = \sum_{j=0}^{r-1} j\alpha_j x^j \tag{2}$$

$$y''_k(x) = \sum_{j=0}^{t-1} j(j-1)\alpha_j x^j \tag{3}$$

where m are the number of interpolation points while r *and* t are the number of collocation points distributed on the step points, and interpolating equation (1) at $x = x_n, x_{n+1}$ and collocating (2) at $x = x_{n+1}, x_{n+2}$ to give the systems of equation and written in matrix form $AX = B$ as

$$\begin{bmatrix} 1 & x_n & x_n^2 & x_n^3 & x_n^4 & x_n^5 \\ 1 & x_{n+1} & x_{n+1}^2 & x_{n+1}^3 & x_{n+1}^4 & x_{n+1}^5 \\ 0 & 1 & 2x_{n+1} & 3x_{n+1}^2 & 4x_{n+1}^3 & 5x_{n+1}^4 \\ 0 & 1 & 2x_{n+2} & 3x_{n+2}^2 & 4x_{n+2}^3 & 5x_{n+2}^4 \\ 0 & 0 & 2 & 6x_{n+1} & 12x_{n+1}^2 & 20x_{n+1}^3 \\ 0 & 0 & 2 & 6x_{n+2} & 12x_{n+2}^2 & 20x_{n+2}^3 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix} = \begin{bmatrix} y_n \\ y_{n+1} \\ f_{n+1} \\ f_{n+2} \\ g_{n+1} \\ g_{n+2} \end{bmatrix} \tag{4}$$

Using Maple 18 software, the inverse of the matrix in (4) is obtained and yields the continuous formulation of the method as:

$$y(x) = \alpha_0(x)y_n + \alpha_1(x)y_{n+1} + h[\beta_0(x)f_n + \beta_1(x)f_{n+1} + \beta_2(x)f_{n+2}] + h^2[\gamma_0(x)g_n + \gamma_1(x)g_{n+1} + \gamma_2(x)]g_{n+2} \tag{5}$$

where the continuous coefficients (5) of the method are given as:

$$\alpha_0(x) = 1 - \frac{120x}{31h} + \frac{180x^2}{31h^2} - \frac{130x^3}{31h^3} + \frac{45x^4}{31h^4} - \frac{6x^5}{31h^5}$$

$$\alpha_1(x) = \frac{120x}{31h} - \frac{180x^2}{31h^2} + \frac{130x^3}{31h^3} - \frac{45x^4}{31h^4} + \frac{6x^5}{31h^5}$$

$$\beta_1(x) = -\frac{64x}{31} + \frac{96x^2}{31h} - \frac{28x^3}{31h^2} + \frac{7x^4}{31h^3} - \frac{3x^5}{31h^4}$$

$$\beta_2(x) = \frac{25x}{31} - \frac{84x^2}{31h} + \frac{102x^3}{31h^2} - \frac{52x^4}{31h^3} + \frac{9x^5}{31h^4}$$

$$\gamma_1(x) = -\frac{46xh}{31} + \frac{96x^2}{31} - \frac{28x^3}{31h} + \frac{7x^4}{31h^2} - \frac{3x^5}{31h^3}$$

$$\gamma_2(x) = \frac{8xh}{31} - \frac{55x^2}{62} + \frac{69x^3}{62h} - \frac{37x^4}{62h^2} + \frac{7x^5}{62h^3} \quad (6)$$

Evaluating (5) at x_n,andx_{n+2} yields the following discrete methods which constitute the new two-step block method.

$$y_n - y_{n+1} = -\frac{h}{15}[8f_{n+1} + 7f_{n+2}] + \frac{h^2}{360}[31g_n + 262g_{n+1} + 55g_{n+2}]$$

$$y_{n+2} + \frac{1}{31}y_n - \frac{32}{31}y_{n+1} = \frac{2h}{31}[8f_{n+1} + 7f_{n+2}] + \frac{2h^2}{31}[2g_{n+1} - g_{n+2}] \quad (7)$$

The method K=2 is of order 4 as a block and has error constant

$$C_6 = \left(-\frac{101}{21600}, \qquad , \qquad \frac{1}{2790} \qquad \right)^T$$

ANALYSIS OF THE NEW METHODS

In this research, we consider the analysis of the newly constructed methods. Its convergence is determined and also regions of absolute stability is plotted below.

CONVERGENCE

The convergence of the new block methods is determined using the approach by Fatunla (1991) and Chollom *et al* (2007) for linear multistep methods, where the block methods are represented in single block, r point multistep method of the form

$$A^{(0)}y_{m+1} = \sum_{i=1}^{k} A^{(i)}y_{m+1} + h\sum_{i=0}^{k} B^{(i)}f_{m+1} + h^2\sum_{i=1}^{k} C^{(i)}g_{m+1} \quad (8)$$

Where h is a fixed mesh size within a block, $A^i, B^i, i = 0,1,2, \ldots .k$ are $r \times r$ identity matrix while $y_m, y_{m-1} and\ y_{m+1}$ are vectors of numerical estimates.

Definition: A numerical method is said to be A-stable if the whole of the left-half plane $\{Z: Re(Z) \leq 0\}$ is contained in the region. $\{Z: Re(Z) \leq 1\}$Where$R(Z)$ is called the stability polynomial of the method (Lambert,1973)

The block method (5) expressed in the form of (6) gives

$$R(z) = z^3 - z^2 = 0$$

Therefore,$|z_1| = 1, z_2 = z_3 = 0$.The block method (6) by definition is A-stable and by Henrici (1962), the block method is convergent

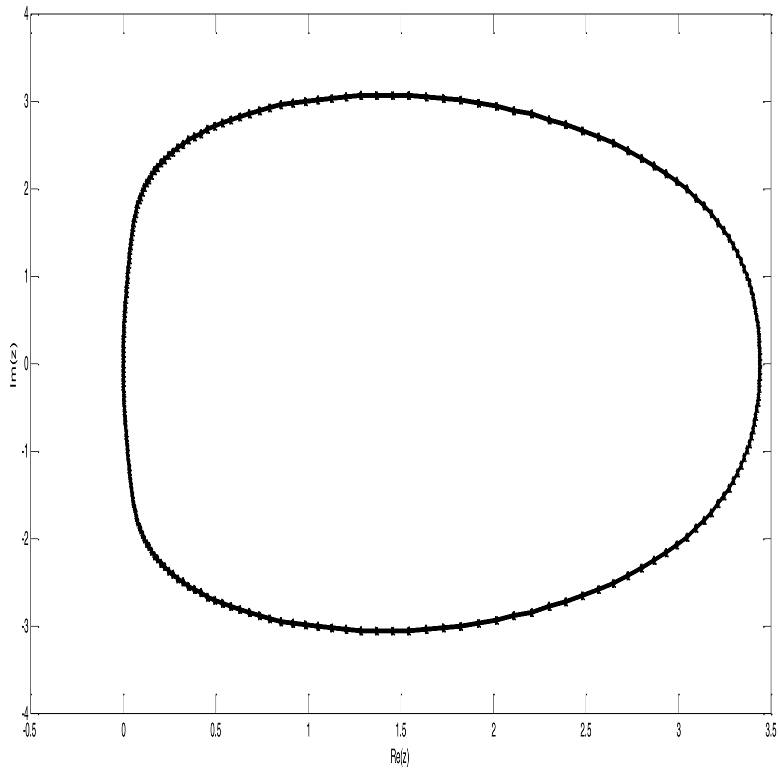

Figure 1. Region of absolute stability of Constructed Method for k=2

NUMERICAL EXPERIMENT AND RESULTS

In this research, the newly constructed continuous conventional second derivative block backward differentiation formulae are applied in block forms for step numbers k = 2 to solve linear stiff system of ordinary differential equations.

Problem 1

$$y_1' = -29998y_1 - 59994y_2$$

$$y_2' = 9999y_1 + 19997y_2$$

Exact $y_1(x) = \left(\frac{1}{9999}\right)(29997e^{-10000x}\text{-}19998e^{-x})$ $\qquad y_1(0) = 1$

$y_2(x) = \text{-}e^{-10000x}+e^{-x}$ $\qquad y_2(0) = 0$

Table 1: Absolute errors of numerical solutions of problem 1 ($h = 0.1$)

x	Theoretical	solutions	New block	method	Absolute	errors
	$y_1(x)$	$y_2(x)$	$y_1(x)$	$y_2(x)$	$y_1(x)$	$y_2(x)$
0.1	1	0	1	0	0	0
0.2	-1.809674836	0.904837418	-2.06012798	0.988321799	2.5045E-01	8.34843E-02
0.3	-1.637461506	0.818730753	-1.61655257	0.811761106	2.09089E-02	6.96964E-03
0.4	-1.481636441	0.740818221	-1.483381994	0.741400069	1.74555E-03	5.81848E-04
0.5	-1.340640092	0.670320046	-1.340494344	0.67027146	1.45748E-04	4.8586E-05
0.6	-1.213061319	0.60653066	-1.213073462	0.606534703	1.21422E-05	4.04341E-06
0.7	-1.097623272	0.548811636	-1.09762223	0.548811284	1.04239E-06	3.51921E-07
0.8	-0.993170608	0.496585304	-0.993170664	0.496585318	5.61236E-08	1.39241E-08
0.9	-0.898657928	0.449328964	-0.898657892	0.449328947	3.57657E-08	1.67081E-08
1.0	-0.813139319	0.40656966	-0.813139291	0.406569645	2.85888E-08	1.43975E-08
1.1	-0.735758882	0.367879441	-0.735758853	0.367879426	2.9428E-08	1.47099E-08
1.2	-0.665742167	0.332871084	-0.665742138	0.332871069	2.96713E-08	1.48401E-08
1.3	-0.602388424	0.301194212	-0.602388394	0.301194197	2.95622E-08	1.47844E-08
1.4	-0.545063586	0.272531793	-0.545063557	0.272531779	2.89127E-08	1.44594E-08
1.5	-0.493193928	0.246596964	-0.4931939	0.24659695	2.83186E-08	1.41621E-08
1.6	-0.44626032	0.22313016	-0.446260293	0.223130146	2.76786E-08	1.38418E-08
1.7	-0.403793036	0.201896518	-0.403793009	0.201896504	2.70486E-08	1.35266E-08
1.8	-0.365367048	0.182683524	-0.365367022	0.182683511	2.59388E-08	1.29715E-08
1.9	-0.330597776	0.165298888	-0.330597752	0.165298876	2.46926E-08	1.23482E-08
2.0	-0.299137238	0.149568619	-0.299137215	0.149568607	2.34575E-08	1.17304E-08
2.1	-0.270670566	0.135335283	-0.270670544	0.135335272	2.24605E-08	1.12318E-08
2.2	-0.244912857	0.122456428	-0.244912835	0.122456418	2.13732E-08	1.0688E-08
2.3	-0.221606317	0.110803158	-0.221606296	0.110803148	2.02973E-08	1.01499E-08
2.4	-0.200517687	0.100258844	-0.200517668	0.100258834	1.934E-08	9.67112E-09
2.5	-0.181435907	0.090717953	-0.181435888	0.090717944	1.82447E-08	9.1234E-09
2.6	-0.164169997	0.082084999	-0.16416998	0.08208499	1.71207E-08	8.56127E-09
2.7	-0.148547156	0.074273578	-0.14854714	0.07427357	1.60982E-08	8.04994E-09
2.8	-0.134411025	0.067205513	-0.13441101	0.067205505	1.52126E-08	7.60707E-09
2.9	-0.121620125	0.060810063	-0.121620111	0.060810055	1.42739E-08	7.13761E-09
3.0	-0.11004644	0.05502322	-0.110046427	0.055023213	1.34023E-08	6.70178E-09

Table 2: Absolute errors of numerical solutions of problem 1 ($h = 0.01$)

x	Theoretical	solutions	New block	method	Absolute	errors
	$y_1(x)$	$y_2(x)$	$y_1(x)$	$y_2(x)$	$y_1(x)$	$y_2(x)$
0.01	-1.000000000	0.000000000	-1.000000000	0.000000000	0.00000E-00	0.00000E-00
0.02	-1.980099667	0.990049834	-2.234159149	1.074736328	2.54059E-01	8.46864E-02
0.03	-1.960397347	0.980198673	-1.93888194	0.973026871	2.15154E-02	7.17180E-03
0.04	-1.940891067	0.970445534	-1.942713131	0.971052888	1.82206E-03	6.07355E-04
0.05	-1.921578878	0.960789439	-1.921424574	0.960738004	1.54304E-04	5.14347E-05
0.06	-1.902458849	0.951229425	-1.902471917	0.95123378	1.30675E-05	4.35584E-06
0.07	-1.883529067	0.941764534	-1.883527961	0.941764165	1.10662E-06	3.68868E-07
0.08	-1.86478764	0.93239382	-1.864787734	0.932393851	9.37579E-08	3.12594E-08
0.09	-1.846232693	0.923116346	-1.846232685	0.923116344	7.90197E-09	2.62821E-09
0.10	-1.827862371	0.913931185	-1.827862371	0.913931186	6.98855E-10	2.37409E-10
0.11	-1.809674836	0.904837418	-1.809674836	0.904837418	3.7268E-11	9.145E-12
0.12	-1.791668271	0.895834135	-1.791668271	0.895834135	5.0665E-11	2.4532E-11
0.13	-1.773840873	0.886920437	-1.773840873	0.886920437	3.6969E-11	1.8555E-11
0.14	-1.756190862	0.878095431	-1.756190862	0.878095431	3.3925E-11	1.6959E-11
0.15	-1.738716471	0.869358235	-1.738716471	0.869358235	3.0283E-11	1.5144E-11
0.16	-1.721415953	0.860707976	-1.721415953	0.860707976	4.0215E-11	2.011E-11
0.17	-1.704287578	0.852143789	-1.704287578	0.852143789	4.7053E-11	2.3529E-11
0.18	-1.687329633	0.843664817	-1.687329633	0.843664817	5.1107E-11	2.5556E-11
0.19	-1.670540423	0.835270211	-1.670540423	0.835270211	5.472E-11	2.7362E-11
0.20	-1.653918268	0.826959134	-1.653918268	0.826959134	4.5668E-11	2.2836E-11
0.21	-1.637461506	0.818730753	-1.637461506	0.818730753	4.5347E-11	2.2676E-11
0.22	-1.621168492	0.810584246	-1.621168492	0.810584246	4.1283E-11	2.0643E-11
0.23	-1.605037596	0.802518798	-1.605037596	0.802518798	2.929E-11	1.4647E-11
0.24	-1.589067205	0.794533603	-1.589067205	0.794533603	4.2928E-11	2.1466E-11
0.25	-1.573255722	0.786627861	-1.573255722	0.786627861	4.1046E-11	2.0525E-11
0.26	-1.557601566	0.778800783	-1.557601566	0.778800783	4.6536E-11	2.327E-11
0.27	-1.542103172	0.771051586	-1.542103172	0.771051586	3.9884E-11	1.9944E-11
0.28	-1.526758989	0.763379494	-1.526758989	0.763379494	3.9506E-11	1.9755E-11
0.29	-1.511567483	0.755783741	-1.511567483	0.755783741	4.6654E-11	2.3329E-11
0.30	-1.496527135	0.748263568	-1.496527135	0.748263568	4.1467E-11	2.0735E-11

Table 3: Absolute errors of numerical solutions of problem 1 ($h = 0.001$)

x	Theoretical	solutions	New block	method	Absolute	errors
	$y_1(x)$	$y_2(x)$	$y_1(x)$	$y_2(x)$	$y_1(x)$	$y_2(x)$
0.001	-0.018725804	0.009362902	-0.018725804	0.009362902	5.4E-14	2.7E-14
0.002	-0.018707087	0.009353544	-0.018707087	0.009353544	5.5E-14	2.7E-14
0.003	-0.01868839	0.009344195	-0.01868839	0.009344195	5.5E-14	2.7E-14
0.004	-0.018669711	0.009334855	-0.018669711	0.009334855	5.6E-14	2.8E-14
0.005	-0.01865105	0.009325525	-0.01865105	0.009325525	5.5E-14	2.7E-14
0.006	-0.018632409	0.009316204	-0.018632409	0.009316204	5.5E-14	2.7E-14
0.007	-0.018613785	0.009306893	-0.018613785	0.009306893	5.4E-14	2.7E-14
0.008	-0.018595181	0.00929759	-0.018595181	0.00929759	5.5E-14	2.7E-14
0.009	-0.018576595	0.009288298	-0.018576595	0.009288298	5.6E-14	2.8E-14
0.010	-0.018558028	0.009279014	-0.018558028	0.009279014	5.5E-14	2.7E-14
0.011	-0.018539479	0.00926974	-0.018539479	0.00926974	5.3E-14	2.7E-14
0.012	-0.018520949	0.009260474	-0.018520949	0.009260474	5.3E-14	2.7E-14
0.013	-0.018502437	0.009251219	-0.018502437	0.009251219	5.3E-14	2.6E-14
0.014	-0.018483944	0.009241972	-0.018483944	0.009241972	5.2E-14	2.6E-14
0.015	-0.018465469	0.009232735	-0.018465469	0.009232735	5.2E-14	2.6E-14
0.016	-0.018447013	0.009223506	-0.018447013	0.009223506	5.3E-14	2.6E-14
0.017	-0.018428575	0.009214288	-0.018428575	0.009214288	5.3E-14	2.6E-14
0.018	-0.018410156	0.009205078	-0.018410156	0.009205078	5.2E-14	2.6E-14
0.019	-0.018391755	0.009195877	-0.018391755	0.009195877	5.3E-14	2.7E-14
0.020	-0.018373372	0.009186686	-0.018373372	0.009186686	5.4E-14	2.7E-14
0.021	-0.018355008	0.009177504	-0.018355008	0.009177504	5.5E-14	2.7E-14
0.022	-0.018336662	0.009168331	-0.018336662	0.009168331	5.5E-14	2.7E-14
0.023	-0.018318335	0.009159167	-0.018318335	0.009159167	5.5E-14	2.7E-14
0.024	-0.018300026	0.009150013	-0.018300026	0.009150013	5.5E-14	2.7E-14
0.025	-0.018281735	0.009140867	-0.018281735	0.009140867	5.2E-14	2.6E-14
0.026	-0.018263462	0.009131731	-0.018263462	0.009131731	5.1E-14	2.6E-14
0.027	-0.018245208	0.009122604	-0.018245208	0.009122604	5.2E-14	2.6E-14
0.028	-0.018226972	0.009113486	-0.018226972	0.009113486	5.2E-14	2.6E-14
0.029	-0.018208754	0.009104377	-0.018208754	0.009104377	5.2E-14	2.6E-14
0.030	-0.018190554	0.009095277	-0.018190554	0.009095277	5.2E-14	2.6E-14

Problem 2

$$y_1' = -2y_1 + y_2 + 2sinx$$

$$y_2' = 998y_1 - 999y_2 + 999(cosx - sinx)$$

Exact $y_{1(x)} = 2e^{-x} + sinx \qquad y_1(0) = 2$

$$y_2(x) = 2e^{-x} + cosx \qquad y_2(0) = 3$$

Table 4: Absolute errors of numerical solutions of problem 2 ($h = 0.1$)

x	Theoretical	solutions	New block	method	Absolute	errors
	$y_1(x)$	$y_2(x)$	$y_1(x)$	$y_2(x)$	$y_1(x)$	$y_2(x)$
0.1	2.000000000	3.000000000	2.000000000	3.000000000	0.00000E+00	0.00000E+00
0.2	1.909508253	2.804679001	1.917268531	2.811245313	7.76028E-03	6.56631E-03
0.3	1.836130837	2.617528084	1.849910892	2.630122552	1.37801E-02	1.25945E-02
0.4	1.777156648	2.43697293	1.795315261	2.453865369	1.81586E-02	1.68924E-02
0.5	1.730058434	2.261701086	1.751053619	2.281370697	2.09952E-02	1.96696E-02
0.6	1.692486858	2.090643881	1.714877766	2.111662339	2.23909E-02	2.10185E-02
0.7	1.662265746	1.922958887	1.684714735	1.944002324	2.24490E-02	2.10434E-02
0.8	1.637388295	1.758012795	1.658663146	1.777863031	2.12749E-02	1.98502E-02
0.9	1.616014019	1.595364638	1.634990162	1.612911335	1.89761E-02	1.75467E-02
1.0	1.596466229	1.434749288	1.612128847	1.448991914	1.56626E-02	1.42426E-02
1.1	1.577229867	1.276061188	1.588675782	1.286110753	1.14459E-02	1.00496E-02
1.2	1.556949527	1.119338289	1.563388749	1.124418754	6.43922E-03	5.08047E-03
1.3	1.53442751	0.964746178	1.535184368	0.964195449	7.56858E-04	5.50729E-04
1.4	1.508621771	0.812562415	1.503135546	0.805832837	5.48622E-03	6.72958E-03
1.5	1.478643658	0.663161071	1.46646865	0.649819368	1.21750E-02	1.33417E-02
1.6	1.443755307	0.516997522	1.424560279	0.496724114	1.91950E-02	2.02734E-02
1.7	1.403366639	0.374593514	1.376933592	0.347181177	2.64330E-02	2.74123E-02
1.8	1.357031859	0.236522554	1.323254112	0.201874392	3.37777E-02	3.46482E-02
1.9	1.304445407	0.103395682	1.26332495	0.061522381	4.11205E-02	4.18733E-02
2.0	1.245437326	-0.024152328	1.197081442	-0.073135963	4.83559E-02	4.89836E-02
2.1	1.179967993	-0.14547627	1.124585148	-0.201355499	5.53828E-02	5.58792E-02
2.2	1.108122223	-0.259933248	1.046017235	-0.322398295	6.21050E-02	6.24650E-02
2.3	1.030102721	-0.366894801	0.961671226	-0.435546431	6.84315E-02	6.86516E-02
2.4	0.9462229	-0.465758334	0.871945147	-0.540114099	7.42778E-02	7.43558E-02
2.5	0.856899087	-0.555957809	0.777333106	-0.635458901	7.95660E-02	7.95011E-02
2.6	0.762642141	-0.636973618	0.678416321	-0.720992295	8.42258E-02	8.40187E-02
2.7	0.664048528	-0.708341597	0.575853673	-0.796189125	8.81949E-02	8.78475E-02
2.8	0.561790906	-0.769661117	0.470371822	-0.86059616	9.14191E-02	9.09350E-02
2.9	0.456608275	-0.820602215	0.362754966	-0.913839607	9.38533E-02	9.32374E-02
3.0	0.349295769	-0.860911725	0.253834305	-0.955631547	9.54615E-02	9.47198E-02

Table 5: Absolute errors of numerical solutions of problem 2 ($h = 0.01$)

x	Theoretical	solutions	New block	method	Absolute	errors
	$y_1(x)$	$y_2(x)$	$y_1(x)$	$y_2(x)$	$y_1(x)$	$y_2(x)$
0.01	2.000000000	3.000000000	2.00000000	3.00000000	0.00000E+00	0.00000E+00
0.02	1.990099501	2.980049668	1.99019516	2.97898723	9.56592E-05	1.06244E-03
0.03	1.980396013	2.960197353	1.980584178	2.959309514	1.88165E-04	8.87840E-04
0.04	1.970886567	2.940441101	1.971165435	2.939626183	2.78867E-04	8.14918E-04
0.05	1.961568212	2.920778985	1.961935881	2.920043257	3.67669E-04	7.35728E-04
0.06	1.952438018	2.901209109	1.952892606	2.90055015	4.54587E-04	6.58959E-04
0.07	1.943493074	2.881729607	1.944032706	2.881145706	5.39632E-04	5.83901E-04
0.08	1.934730487	2.86233864	1.9353533	2.861828041	6.22813E-04	5.10599E-04
0.09	1.926147387	2.843034399	1.926851526	2.842595362	7.04140E-04	4.39037E-04
0.10	1.91774092	2.823815104	1.918524541	2.823445898	7.83622E-04	3.69206E-04
0.11	1.909508253	2.804679001	1.910369521	2.804377907	8.61268E-04	3.01094E-04
0.12	1.901446571	2.785624369	1.902383661	2.785389677	9.37090E-04	2.34692E-04
0.13	1.893553081	2.766649509	1.894564177	2.766479522	1.01110E-03	1.69988E-04
0.14	1.885825004	2.747752756	1.886908301	2.747645784	1.08330E-03	1.06971E-04
0.15	1.878259585	2.728932467	1.879413286	2.728886835	1.15370E-03	4.56316E-05
0.16	1.870854085	2.710187031	1.872076404	2.710201072	1.22232E-03	1.40415E-05
0.17	1.863605785	2.691514861	1.864894945	2.69158692	1.28916E-03	7.20589E-05
0.18	1.856511982	2.6729144	1.857866217	2.673042831	1.35423E-03	1.28431E-04
0.19	1.849569996	2.654384116	1.850987549	2.654567285	1.41755E-03	1.83169E-04
0.20	1.842777163	2.635922503	1.844256286	2.636158787	1.47912E-03	2.36284E-04
0.21	1.836130837	2.617528084	1.837669793	2.617815869	1.53896E-03	2.87785E-04
0.22	1.829628392	2.599199407	1.831225454	2.59953709	1.59706E-03	3.37684E-04
0.23	1.823267219	2.580935045	1.824920669	2.581321036	1.65345E-03	3.85991E-04
0.24	1.817044729	2.5627336	1.818752859	2.563166317	1.70813E-03	4.32717E-04
0.25	1.810958349	2.544593697	1.812719461	2.545071569	1.76111E-03	4.77872E-04
0.26	1.805005525	2.526513988	1.806817932	2.527035456	1.81241E-03	5.21468E-04
0.27	1.799183723	2.50849315	1.801045746	2.509056665	1.86202E-03	5.63515E-04
0.28	1.793490425	2.490529885	1.795400396	2.491133909	1.90997E-03	6.04024E-04
0.29	1.787923131	2.472622921	1.789879392	2.473265926	1.95626E-03	6.43005E-04
0.30	1.78247936	2.454771011	1.784480262	2.455451479	2.00090E-03	6.80469E-04

Table 6: Absolute errors of numerical solutions of problem 2 ($h = 0.001$)

x	Theoretical	solutions	New block	method	Absolute	errors
	$y_1(x)$	$y_2(x)$	$y_1(x)$	$y_2(x)$	$y_1(x)$	$y_2(x)$
0.001	-0.980417795	-0.022651361	-0.980465903	-0.022052997	4.81080E-05	5.98363E-04
0.002	-0.980477389	-0.021670913	-0.980524523	-0.021072171	4.71340E-05	5.98742E-04
0.003	-0.980535965	-0.020690406	-0.980582125	-0.020091287	4.61598E-05	5.99119E-04
0.004	-0.980593523	-0.019709841	-0.980638709	-0.019110345	4.51856E-05	5.99496E-04
0.005	-0.980650063	-0.018729219	-0.980694275	-0.018129347	4.42114E-05	5.99873E-04
0.006	-0.980705586	-0.017748542	-0.980748823	-0.017148293	4.32372E-05	6.00248E-04
0.007	-0.98076009	-0.016767809	-0.980802353	-0.016167185	4.22630E-05	6.00623E-04
0.008	-0.980813576	-0.015787022	-0.980854865	-0.015186024	4.12887E-05	6.00998E-04
0.009	-0.980866045	-0.014806182	-0.980906359	-0.01420481	4.03144E-05	6.01372E-04
0.010	-0.980917495	-0.01382529	-0.980956835	-0.013223545	3.93401E-05	6.01745E-04
0.011	-0.980967927	-0.012844347	-0.981006293	-0.01224223	3.83657E-05	6.02118E-04
0.012	-0.981017342	-0.011863354	-0.981054733	-0.011260865	3.73914E-05	6.02490E-04
0.013	-0.981065738	-0.010882313	-0.981102155	-0.010279452	3.64170E-05	6.02861E-04
0.014	-0.981113116	-0.009901223	-0.981148559	-0.009297992	3.54426E-05	6.03232E-04
0.015	-0.981159476	-0.008920087	-0.981193944	-0.008316485	3.44682E-05	6.03602E-04
0.016	-0.981204818	-0.007938905	-0.981238312	-0.007334933	3.34937E-05	6.03971E-04
0.017	-0.981249142	-0.006957678	-0.981281661	-0.006353337	3.25193E-05	6.04340E-04
0.018	-0.981292448	-0.005976407	-0.981323993	-0.005371698	3.15448E-05	6.04709E-04
0.019	-0.981334735	-0.004995093	-0.981365306	-0.004390017	3.05703E-05	6.05076E-04
0.020	-0.981376005	-0.004013738	-0.981405601	-0.003408294	2.95958E-05	6.05443E-04
0.021	-0.981416256	-0.003032341	-0.981444878	-0.002426532	2.86213E-05	6.05810E-04
0.022	-0.98145549	-0.002050905	-0.981483136	-0.00144473	2.76468E-05	6.06175E-04
0.023	-0.981493705	-0.001069431	-0.981520377	-0.00046289	2.66723E-05	6.06541E-04
0.024	-0.981530902	-8.79184E-05	-0.9815566	0.000518987	2.56977E-05	6.06905E-04
0.025	-0.981567081	0.000893631	-0.981591804	0.0015009	2.47232E-05	6.07269E-04
0.026	-0.981602242	0.001875215	-0.98162599	0.002482848	2.37487E-05	6.07632E-04
0.027	-0.981636384	0.002856835	-0.981659158	0.00346483	2.27741E-05	6.07995E-04
0.028	-0.981669509	0.003838488	-0.981691308	0.004446845	2.17995E-05	6.08357E-04
0.029	-0.981701615	0.004820173	-0.98172244	0.005428892	2.08250E-05	6.08718E-04
0.030	-0.981732703	0.005801891	-0.981752554	0.00641097	1.98504E-05	6.09079E-04

CONCLUSION

The multistep collocation approach was used in the derivation of proposed uniform order four conventional block linear multistep methods for numerical solution of stiff systems of ordinary differential equations for the step number $k = 2$. Our new block scheme of uniform order 4 performed well with marginal absolute error constants of first order stiff Ordinary differential equation (ODE) problem solved. It was observed that as the step size approaches zero the better the performance of the method and numerical results obtained in Table 1, 2 ,3,4,5 and 6 using the new block method compare favourably with theoretical results. The numerical results reveal the accuracy of the newly constructed method.

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