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Counting Distinct Fuzzy Subgroups of some Alternating Groups

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Abstract
This research was designed to extend the counting of distinct fuzzy subgroups to . The equivalence relation applied in our computation can be found in existing literatures. Computer Algebra Program GAP(Group Algorithm Programming) was applied in counting the subgroups of the alternating groups. Lattice subgroups diagrams of alternating groups were used in our computation. Literature had revealed that number of fuzzy subgroups of was 402, however, our computation has shown that has 408 distinct fuzzy subgroups. We were also able to show that has 30548 distinct fuzzy subgroups.

Keywords: Alternating groups; Fuzzy subgroups; Equivalence relation; GAP

1. Introduction
Distinguishing objects in classical set theory is not difficult, it can be determined with certainty whether or not an object belongs to a particular set. This is because, classical set theory depicts an ideal situation. For example, Nigeria is a member of the set of Africa Union countries while Brazil is not. Such sets are called crisp sets. However, in our day to day life activities, everything can not be categorised into well defined sets. The quest to classify observations of this forms in mathematical realm emanated the development of fuzzy set.

The origin of this lively branch of mathematics can be traced to a diversity of famous thinkers which includes Plato, Stanisaw Jakowski, Georg Wilhelm Friedrich Hegel, Karl Marx, Friedrich Engels, Friedrich Nietzsche, Jan ukasiewicz, Alfred Tarski and Donald Knuth, see Priyanka et al (2010) .

Counting fuzzy subgroups of alternating groups is a fundamental problem of fuzzy group theory. Researchers had studied and counted fuzzy subgroups of alternating groups A_3, A_4 *and* A_5, see, Suleiman and Abd Ghafur (2011) and Ogiugo and EniOluwafe (2017) . However, the number of fuzzy subgroups of A_6 had not been computed. Thus, in this paper we extend the research to counting fuzzy subgroups of A_6 and give a moderation of A_5 from 402 distinct subgroups to 408 distinct fuzzy subgroups.

The paper is divided into four sections. Section one gives the introduction and literature review as above. Section two gives some preliminary results, definitions and an overview of the equivalence as defined by Suleiman and Abd Ghafur (2011). In section three we give

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modification to counting distinct fuzzy subgroups of A_5, and also count distinct fuzzy subgroups A_6 and we obtained 30548 distinct fuzzy subgroups. Finally, in section four, we give conclusion on our result.

2. Preliminaries

In this section, we give some definitions, preliminary results and an overview of equivalence relations introduced as defined by Suleiman and Abd Ghafur (2011, 2012).

2.1 Definition: Let G be a group, a function μ from G into $[0,1]$ is called afuzzy subgroup of G if $\mu(xy) \geq min\{\mu(x), \mu(y)\}$, $\mu(x^{-1}) \geq \mu(x), \forall x \in G$.

2.2 Theorem: Sulaiman and Abd Ghafur (2011): A Function $\mu : G \rightarrow [0,1]$ is a fuzzy subgroup of G if thre is a chain of subgroups $P_1 \leq P_2 \leq P_3 \leq ... \leq P_n = G$, in subgroup lattice of G which can be written as;

$$\mu(x) = \begin{cases} \theta_1, & x \in P_1 \\ \theta_1, & x \in P_2 \setminus P_1 \\ \theta_n, & x \in P_n \setminus P_{n-1} \end{cases} \quad (1)$$

Without any equivalence relations on fuzzy subgroups of a group *G*, the number of fuzzy subgroups is infinite even for the trivial group *{e}*. So we apply the equivalence relation defined by Sulaiman and Abd Ghafur (2011) to count all distinct fuzzy subgroups of alternating group A_6.

2.3 An overview of the equivalence relation by Sulaiman and Abd Ghafur (2011)

Let μ and γ be fuzzy subgroups of G of the following forms;

$$\mu(x) = \begin{cases} \theta_1, & x \in P_1 \\ \theta_2, & x \in P_2 \setminus P_1 \\ \theta_n, & x \in P_n \setminus P_{n-1} \end{cases}, \quad \gamma(x) = \begin{cases} \delta_1, & x \in M_1 \\ \delta_2, & x \in M_2 \setminus M_1 \\ \delta_m, & x \in M_m \setminus M_{m-1} \end{cases} \quad (2)$$

Then we say that μ and γ are equivalent and write $\mu \sim \gamma$ if $m = n$, and $P_i = M_i \ \forall i = \{1, \ldots, m\}$.

2.4 Lemma: The number of fuzzy subgroups of G is equal to the number of chains on the subgroups lattice of G.

3. Results and Discussion

In this section, we present results on distinct fuzzy subgroups of A_6 and discuss on the modification of distinct fuzzy subgroups of A_5.

3.1 Counting Distinct Fuzzy Subgroups of A_5

We recall that A_5 has sixty(60) elements generated as follows using GAP ;

Let $A_5 = \{id = e, \psi_1,.., \psi_5, v_1,.., v_7, \tau_1,.., \tau_{10}, \sigma_1,.., \sigma_6, \eta_1,.., \eta_6, \varphi_1,.., \varphi_{10}, \delta_1,.., \delta_{15}\}$ with:

$\delta_1 = (23)(45)$, $\delta_2 = (24)(35)$, $\delta_3 = (25)(34)$, $\delta_4 = (12)(45)$, $\delta_5 = (12)(34)$, $\delta_6 = (12)(35)$, $\delta_7 = (13)(45)$, $\delta_8 = (13)(24)$, $\delta_9 = (13)(25)$, $\delta_{10} = (14)(35)$, $\delta_{11} = (14)(23)$, $\delta_{12} = (14)(25)$,

δ_{13} = (15)(34), δ_{14} = (15)(23), δ_{15} = (15)(24), φ_1 = (345), φ_2 = (234), φ_3 = (253), φ_4 = (245), φ_5 = (123), φ_6 = (142), φ_7 = (125), φ_8 = (134), φ_9 = (153), φ_{10} = (145), η_1 = (23)(45),(24)(35), η_2 = (12)(45),(14)(25), η_3 = (12)(34),(13)(24), η_4 = (15)(23),(13)(25), η_5 = (15)(34),(14)(35), η_6 = (12)(45),(14)(25), σ_1 = (12345), σ_2 = (15243), σ_3 = (12453), σ_4 = (14523), σ_5 = (14235), σ_6 = (12534), τ_1 = (12)(45),(345), τ_2 = (23)(45),(123), τ_3 = (15)(23),(145), τ_4 = (15)(34),(234), τ_5 = (14)(35),(253), τ_6 = (13)(25),(245), τ_7 = (24)(35),(142), τ_8 = (13)(24),(153), τ_9 = (12)(34),(125), τ_{10} = (14)(25),(134), v_1 = (15)(24),(14)(35), v_2 = (12)(35),(14)(25), v_3 = (13)(25),(15)(34), v_4 = (12)(34),(15)(23), v_5 = (13)(24),(12)(45), v_6 = (14)(23),(13)(45), v_7 = (14)(35),(15)(24), ψ_1 = (345),(24)(35), ψ_2 = (145),(14)(35), ψ_3 = (234),(13)(24), ψ_4 = (123),(13)(25), ψ_5 = (125),(14)(25).

There are 57 non tivial subgroups of A_5 namely;

Subgroups of order two(2)

$P_1 = \{i,\delta_1\}$, $P_2 = \{i,\delta_2\}$, $P_3 = \{i,\delta_3\}$, $P_4 = \{i,\delta_4\}$, $P_5 = \{i,\delta_5\}$, $P_6 = \{i,\delta_6\}$, $P_7 = \{i,\delta_7\}$, $P_8 = \{i,\delta_8\}$, $P_9 = \{i,\delta_9\}$, $P_{10} = \{i,\delta_{10}\}$, $P_{11} = \{i,\delta_{11}\}$, $P_{12} = \{i,\delta_{12}\}$, $P_{13} = \{i,\delta_{13}\}$, $P_{14} = \{i,\delta_{14}\}$, $P_{15} = \{i,\delta_{15}\}$

Subgroups of order three(3)

$Q_{16} = \{i, \varphi_1, \varphi_2\}$, $Q_{17} = \{i, \varphi_2, \varphi_3\}$, $Q_{18} = \{i, \varphi_3, \varphi_4\}$, $Q_{19} = \{i, \varphi_4, \varphi_5\}$, $Q_{20} = \{i, \varphi_5, \varphi_6\}$, $Q_{21} = \{i, \varphi_6, \varphi_7\}$, $Q_{22} = \{i, \varphi_7, \varphi_8\}$, $Q_{23} = \{i, \varphi_8, \varphi_9\}$, $Q_{24} = \{i, \varphi_9, \varphi_{10}\}$, $Q_{25} = \{i, \varphi_9, \varphi_{10}\}$

Subgroups of order four(4)

$R_{26} = \{i, \eta_1, \eta_2, \delta_1\}$, $R_{27} = \{i, \eta_3, \eta_4, \delta_2\}$, $R_{28} = \{i, \eta_5, \eta_6, \delta_3\}$, $R_{29} = \{i, \eta_2, \eta_5, \delta_4\}$, $R_{30} = \{i, \eta_3, \eta_1, \delta_5\}$

Subgroups of order five(5)

$S_{31} = \{i, \sigma_1, \sigma_2, \sigma_3, \sigma_4\}$, $S_{32} = \{i, \sigma_5, \sigma_6, \sigma_3, \sigma_4\}$, $S_{33} = \{i, \sigma_1, \sigma_2, \sigma_5, \sigma_6\}$, $S_{34} = \{i, \sigma_1, \sigma_3, \sigma_5, \sigma_4\}$, $S_{35} = \{i, \sigma_2, \sigma_4, \sigma_6, \sigma_1\}$, $S_{36} = \{i, \sigma_3, \sigma_2, \sigma_1, \sigma_4\}$

Subgroups of order six(6)

$T_{37} = \{i, \tau_1, \tau_2, \varphi_1, \varphi_2, \delta_1\}$, $T_{38} = \{i, \tau_3, \tau_4, \varphi_3, \varphi_4, \delta_2\}$, $T_{39} = \{i, \tau_5, \tau_6, \varphi_5, \varphi_6, \delta_3\}, \cdots,$ T46 = $\{i, \tau 8, \tau 10, \varphi 8, \varphi 10, \delta 10\}$

Subgroups of order ten(10)

$U_{47} = \{i, \delta_1, \delta_2, \delta_3, \delta_4, \sigma_1, \sigma_2, \sigma_3, v_1\}$, $U_{48} = \{i, \delta_1, \delta_2, \delta_3, \delta_4, \sigma_1, \sigma_2, \sigma_3, v_1\}, \cdots,$ $U_{52} = \{i, \delta_1, \delta_2, \delta_3, \delta_4, \sigma_1, \sigma_2, \sigma_3, v_1\}$

Subgroups of order twelve(12)

$W_{53} = \{i, \delta_1, \delta_2, \delta_3, \delta_4, \delta_5, \delta_6, \eta_1, \eta_2, \eta_3, \eta_4, \psi_1\}, \cdots, W_{57} = \{i, \delta_{10}, \delta_{11}, \delta_{12}, \delta 13, \delta 14, \delta 15, \eta 2, \eta 3, \eta 5, \eta 6, \psi 5\}$

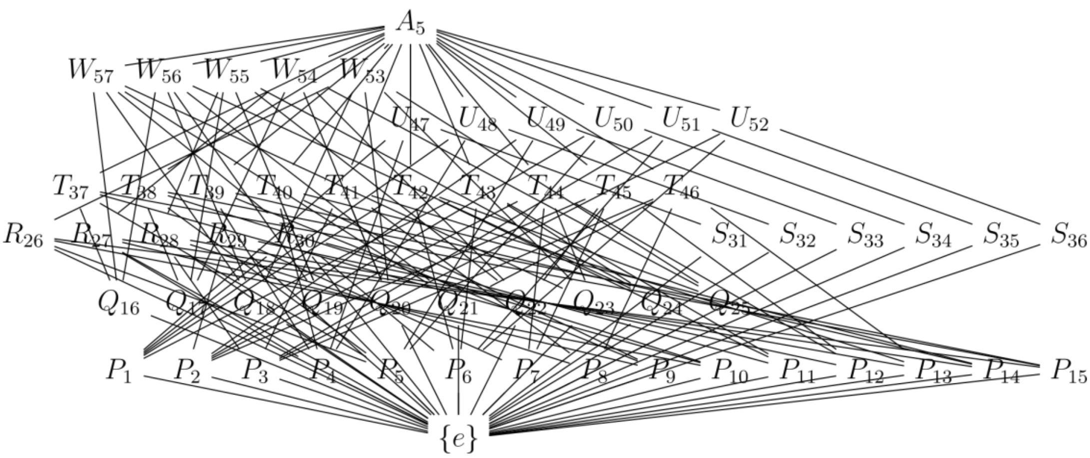

Figure 1: Subgroup Lattice of A_5

We will identify μ according to $P_1(\mu)$. The number of fuzzy subgroup of A_5 with $P_1(\mu) = W$ is denoted by $o(P_1 = W)$.

Note that every subgroup of A_5 can be chosen to be $P_1(\mu)$. If we have $P_1(\mu) = W_{57}$, $P_1(\mu) = W_{57}$, $P_1(\mu) = W_{57}$, $P_1(\mu) = W_{57}$, $P_1(\mu) = W_{53}$, then we only have one fuzzy subgroup of A_5.
If $P_1(\mu) = U_{47}$, we only have one fuzzy subgroup of A_5, that is $\mu_1(x) = \theta_1 \forall x \in A_5$. If $P_1(\mu) = A_5$.
If we have one fuzzy subgroup for $P_1(\mu) = U_{47}$, $P_1(\mu) = U_{48}$, $P_1(\mu) = U_{49}$, $P_1(\mu) = U_{50}$, $P_1(\mu) = U_{51}$, $P_1(\mu) = U_{52}$. Thus, $o(P_1 = A_5) = o(P_1 = U_{47}) = o(P_1 = U_{48}) = o(P_1 = U_{49}) = o(P_1 = U_{50}) = o(P_1 = U_{51}) = o(P_1 = U_{52}) = 1$, then we have one chain, which is $U_{47} < A_5$.
If $P_1(\mu) = R_{26}$, then we have two chains, those are $R_{26} < A_5$ and $R_{26} < W_{57} < A_5$. Therefore, we get two fuzzy subgroups of A_5 with $P_1 = R_{26}$, those are

$$\mu_{22}(x) = \begin{cases} \theta_1, & x \in R_{26} \\ \theta_2, & x \in A_5 \setminus R_{26} \end{cases}, \quad \mu_{22}(x) = \begin{cases} \theta_1, & x \in R_{26} \\ \theta_2, & x \in W_{57} \setminus R_{26} \\ \theta_3, & x \in A_5 \setminus R_{26} \end{cases} \tag{3}$$

Similarly, we have two fuzzy subgroups for $P_1(\mu) = R_{27}$, $P_1(\mu) = R_{28}$, $P_1(\mu) = R29$, $P1(\mu) = R30$.
If $P_1(\mu) = S_{31}$, then we also have two chains, those are $S_{31} < A_5$ and $S_{31} < U_{47} < A_5$. Therefore, we get two fuzzy subgroups of A_5 with $P_1(\mu) = S_{32}$ and we have the same number for $P_1(\mu) = S_{33}$, $P_1(\mu) = S_{34}$, $P_1(\mu) = S_{35}$, $P_1(\mu) = S_{36}$. If $P_1(\mu) = Q_{16}$, then we have four chains, those are $Q_{16} < A_5$, $Q_{16} < T_{37} < A_5$, $Q_{16} < W_{56} < A_5$ and $Q_{16} < W_{57} < A_5$. Therefore, we get four fuzzy subgroups of A_5 with $P_1(\mu) = Q_{16}$ and we have the same number for $P_1(\mu) = Q_{17}$, $P_1(\mu) = Q_{18}$, $P_1(\mu) = Q_{19}$, $P_1(\mu) = Q_{20}$, $P_1(\mu) = Q_{21}$, $P_1(\mu) = Q_{22}$, $P_1(\mu) = Q_{23}$, $P_1(\mu) = Q_{24}$, $P_1(\mu) = Q_{25}$.

Similarly, we have;
Eight fuzzy subgroups of A_5 for $P_1(\mu) = P_1$, $P_1(\mu) = P_2$, $P_1(\mu) = P_3$, $P_1(\mu) = P_4$, $P_1(\mu) = P_5$, $P_1(\mu) = P_6$, $P_1(\mu) = P_7$, $P_1(\mu) = P_8$, $P_1(\mu) = P_9$, $P_1(\mu) = P_{10}$, $P_1(\mu) = P_{11}$, $P_1(\mu) = P_{12}$, $P_1(\mu) = P_{13}$, $P_1(\mu) = P_{14}$ and $P_1(\mu) = P_{15}$. Finally, we have 204 fuzzy subgroups of A_5 for $P_1(\mu) = \{e\}$.

Thus, the total number of fuzzy subgroups of A_5 is 22 + 12 + 10 + 40 + 120 +204 = **408**. This value modifies the 402 value obtained by Ogiugo and EniOluwafe (2017).

3.2 Number of Fuzzy Subgroups of A_6

3.3 Theorem: The number of fuzzy subgroups of alternating group A_6 is 30538.
Proof: Let $A_6 = \{id = e, \zeta_1, \zeta_2, \zeta_3, \zeta_4, \cdots, \zeta_{14}, \imath_1, \imath_2, \imath_3, \imath_4, \cdots, \imath_{12}, \xi_1, \xi_2, \xi_3, \xi_4, \cdots, \xi_{30}, \%_1, \%_2, \%_3, \%_4, \cdots, \%_{10}, \phi_1, \phi_2, \phi_3, \phi_4, \cdots, \phi_{30}, \psi_1, \psi_2, \psi_3, \psi_4, \cdots, \psi_{36}, \varphi_1, \varphi_2, \varphi_3, \varphi_4, \cdots, \varphi_{10}, \sigma_1, \sigma_2, \sigma_3, \sigma_4, \cdots, \sigma_{45}, \varsigma_1, \varsigma_2, \varsigma_3, \varsigma_4, \cdots, \varsigma_{120}, \vartheta_1, \vartheta_2, \vartheta_3, \vartheta_4, \cdots, \vartheta_{36}, \nu_1, \nu_2, \nu_3, \nu_4, \cdots, \nu_{30}, \eta_1, \eta_2, \eta_3, \eta_4, \cdots, \eta_{45}, \gamma_1, \gamma_2, \gamma_3, \gamma_4, \cdots, \gamma_{40}, \delta_1, \delta_2, \delta_3, \delta_4, \cdots, \delta_{45}\}$

From GAP, we have the listed the elements as $\delta_1 = (34)(56)$, $\delta_2 = (35)(46)$, $\delta_3 = (36)(45)$, $\delta_4 = (23)(56)$, $\cdots$, $\delta_{45} = (16)(25)$, $\gamma_1 = (456)$, $\gamma_2 = (354)$, $\gamma_3 = (346)$, $\gamma_4 = (365)$, $\cdots$, $\gamma_{40} = (165)(234)$, $\nu_1 = (162,5)(34)$, $\nu_2 = (1524)(36)$, $\nu_3 = (1426)(35)$, $\nu_4 = (1324)(56)$, $\cdots$, $\nu_{30} = (1564)(23)$, $\vartheta_1 = (24635)$, $\vartheta_2 = (23465)$, $\vartheta_3 = (25436)$, $\vartheta_4 = (26453)$, $\cdots$, $\vartheta_{36} = (1,3645)$, $\varsigma_1 = (456),(23)(45)$, $\varsigma_2 = (456),(12)(45)$, $\varsigma_3 = (456),(13)(45)$, $\varsigma_4 = (234),(34)(56)$, $\cdots$, $\varsigma_{120} = (26)(35),(135)(264)$,
$\sigma_1 = (3,6)(4,5),(1,2)(3,4)$, $\sigma_2 = (35)(46),(12)(36)$, $\sigma_3 = (34)(56),(12)(35)$, $\sigma_4 = (14)(23),(12)(56)$, $\cdots$, $\sigma_{45} = (15)(34),(14)(26)$, $\varphi_1 = (132)(456),(123)(456)$, $\varphi_2 = (126)(354),(126)(345)$, $\varphi_3 = (152)(346),(125)(346)$, $\varphi_4 = (124)(365),(124)(356)$, $\cdots$, $\varphi_{10} = (143)(256),(134)(256)$, $\psi_1 = (24)(36),(25)(46)$, $\psi_2 = (26)(45),(23)(46)$, $\psi_3 = (14)(36),(16)(35)$, $\psi_4 = (13)(46),(14)(35)$, $\cdots$, $\psi_{36} = (15)(23),(13)(56)$, $\phi_1 = (364),(34)(56)$, $\phi_2 = (246),(25)(46)$, $\phi_3 = (465),(16)(45)$, $\phi_4 = (243),(23)(45)$, $\cdots$, $\phi_{30} = (152)(364),(14)(26)$, $\%_1 = (132)(456),(13)(56),(13)(45)$, $\%_2 = (165)(243),(16)(24),(15)(24)$, $\%_3 = (143)(256),(13)(25),(13)(26)$, $\%_4 = (126)(354),(26)(35),(12)(35)$, $\cdots$, $\%_{10} = (163)(254),(13)(24),(24)(36)$, $\xi_1 = (456),(12)(3465)$, $\xi_2 = (165),(1456)(23)$, $\xi_3 = (245),(13)(2564)$, $\xi_4 = (234),(1243)(56)$, $\cdots$, $\xi_{30} = (164)(253),(1236)(45)$, $\imath_1 = (13)(56),(1526)(34)$, $\imath_2 = (16)(24),(1362)(45)$, $\imath_3 = (13)(25),(16)(2453)$, $\imath_4 = (26)(35),(15)(2463)$, $\cdots$, $\imath_{12} = (13)(24),(1435)(26)$, $\zeta_1 = (23)(45),(346)$, $\zeta_2 = (15)(34),(456)$, $\zeta_3 = (12)(45),(165)$, $\zeta_4 = (12)(35),(234)$, $\cdots$, $\zeta_{14} = (12)(36),(136)(245)$

There are 499 non trivial subgroups of A_6 namely:

Subgroups of order two (2)
$B_1 = \{i, \delta_1\}$, $B_2 = \{i, \delta_2\}$, $B_3 = \{i, \delta_3\}$, $B_4 = \{i, \delta_4\}$, $\cdots$, $B_{45} = \{i, \delta_{45}\}$
Subgroups of order three
$C_{46} = \{i, \gamma_1, \gamma_2\}$, $C_{47} = \{i, \gamma_3, \gamma_4\}$, $C_{48} = \{i, \gamma_5, \gamma_6\}$, $C_{49} = \{i, \gamma_6, \gamma_7\}$, $\cdots$, $C_{85} = \{i, \gamma_{39}, \gamma_{40}\}$

Subgroups of order four (4)
$D_{86} = \{i, \delta_1, \delta_2, \eta_1\}$, $D_{87} = \{i, \delta_3, \delta_4, \eta_2\}$, $D_{88} = \{i, \delta_5, \delta_6, \eta_3\}$, $D_{89} = \{i, \delta_7, \delta_8, \eta_4\}$, $\cdots$, $D_{129} = \{i, \delta_{44}, \delta_{45}, \eta_{45}\}$,

Klein-four subgroups
$V_1 = \{i, \sigma_1, \phi_1, \phi_2,\}$, $V_2 = \{i, \sigma_2, \phi_3, \phi_4,\}$, $V_3 = \{i, \sigma_3, \phi_5, \phi_6,\}$, $\cdots$, $V30 = \{i, \sigma 45, \phi 29, \phi 30\}$

Subgroups of order five (5)
$E_{160} = \{i, \vartheta_1, \vartheta_2, \vartheta_3, \vartheta_4\}$, $E_{161} = \{i, \vartheta_5, \vartheta_6, \vartheta_7, \vartheta_8\}$, $E_{162} = \{i, \vartheta_9, \vartheta_{10}, \vartheta_{11}, \vartheta_{12}\}$, $\cdots$, $E_{195} = \{i, \vartheta_{33}, \vartheta_{34}, \vartheta_{35}, \vartheta_{36}\}$

Subgroups of order six (6)
$F_{196} = \{i, \delta_1, \delta_2, \gamma_1, \gamma_2, \varsigma_1\}$, $F_{197} = \{i, \delta_3, \delta_4, \gamma_3, \gamma_4, \varsigma_2\}$, $F_{198} = \{i, \delta_5, \delta_6, \gamma_5, \gamma_6, \varsigma_3\}$, $\cdots$, $F_{315} = \{i, \delta_{44}, \delta_{45}, \gamma_{44}, \gamma_{45}, \varsigma_{120}\}$

Subgroups of order eight (8)
$G_{316} = \{i,\delta_1,\delta_2,\eta_1,\eta_2,\sigma_1,\sigma_2,\sigma_3\}$, $G_{317} = \{i,\delta_3,\delta_4,\eta_3,\eta_4,\sigma_4,\sigma_5,\sigma_6\}$,
$G_{318} = \{i,\delta 5,\delta_6,\eta_5,\eta_6,\sigma_7,\sigma_8,\sigma_9\}, \cdots, G_{360} = \{i,\delta_{44},\delta_{45},\eta_{44},\eta_{45},\sigma_{43},\sigma_{44},\sigma_{45}\}$

Subgroups of order nine (9)
$H_{361} = \{i, \gamma_1, \gamma_2, \gamma_3, \varphi_1, \varphi_2, \varphi_3, \varphi_4, \varphi_5\}$, $H_{362} = \{i, \gamma_4, \gamma_5, \gamma_6, \varphi_6, \varphi_7, \varphi_8, \varphi_9, \varphi_{10}\}$,
$H_{363} = \{i, \gamma_7, \gamma_8, \gamma_9, \varphi_4, \varphi_2, \varphi_6, \varphi_8, \varphi_{10}\}, \cdots, H_{370} = \{i, \gamma_{38}, \gamma_{39}, \gamma_{40}, \varphi_1, \varphi_3, \varphi_5, \varphi_7, \varphi_9\}$

Subgroups of order ten (10)
$I_{371} = \{i, \vartheta_1, \vartheta_2, \vartheta_3, \vartheta_4, \psi_1, \psi_2, \psi_3, \psi_4, \psi_5\}$, $I_{372} = \{i, \vartheta_5, \vartheta_6, \vartheta_7, \vartheta_8, \psi_6, \psi_7, \psi_8, \psi_9, \psi_{10}\}$,
$I_{373} = \{i, \vartheta_9, \vartheta_{10}, \vartheta_{11}, \vartheta_{12}, \psi_{11}, \psi_{12}, \psi_{13}, \psi_{14}, \psi_{15}\}, \cdots,$
$I_{406} = \{i, \vartheta_{33}, \vartheta_{34}, \vartheta_{35}, \vartheta_{36}, \psi_{32}, \psi_{33}, \psi_{34}, \psi_{35}, \psi_{36}\}$
Similarly,

Subgroups of order twelve (12)
Named as J_i, $\forall i = 407, 408, 409, 410, ..., 436$.
Subgroups of order eighteen (18)
Named as K_i, $\forall i = 437, 438, 439, 440, ..., 446$.

Subgroups of order twenty-four (24) Named as L_i, $\forall i = 447, 448, 449, 450, ..., 476$.

Subgroups of order thirty-six (36)
Named as M_i, $\forall i = 477,478,479,480,....,486$.

Subgroups of order sixty (60)
Named as N_i, $\forall i = 487,488,489,490,....,499$.
We count the number of fuzzy subgroups of A_6. Let μ be a fuzzy subgroup of A_6, we will also identify μ according to $P_1(\mu)$.
The number of fuzzy subgroup of A_6 with $P_1(\mu) = N$ denoted by $o(P_1 = N)$. Note that every subgroup of A_6 can be chosen to be $P_1(\mu)$.

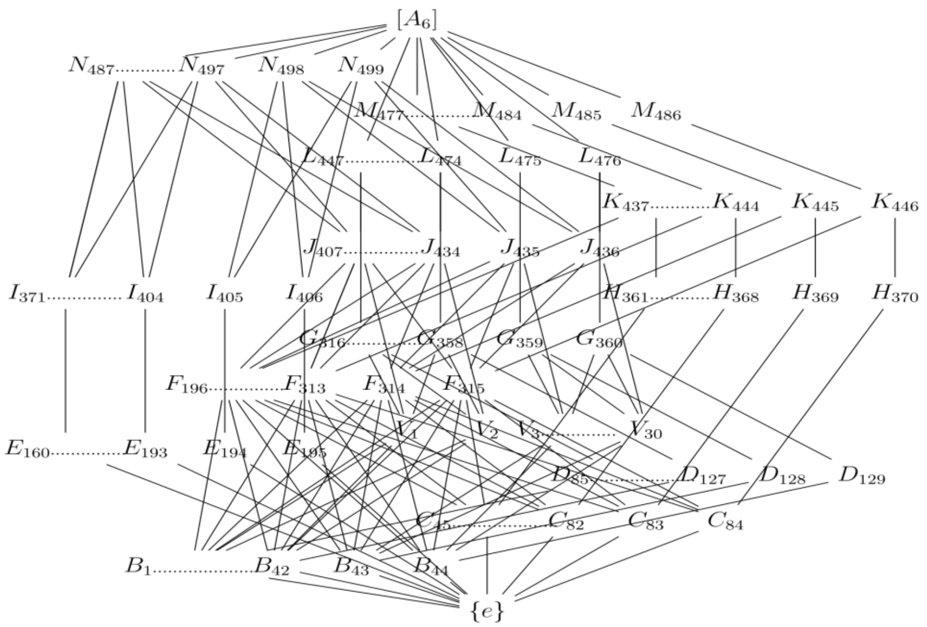

Figure 2: Subgroup Lattice of A_6

If we have $P_1(\mu) = N_{499}$, then we only have one fuzzy subgroup of A_6, that is $\mu_1(x) = \theta_1 \forall x \in A_6$. If $P_1(\mu) = A_6$. Similarly,we have one fuzzy subgroup for $P_1(\mu) = N_{498}$, $P_1(\mu) = N_{497}$, $P_1(\mu) = N_{496}$, $P_1(\mu) = N_{495}$, $P_1(\mu) = N_{494}$, $P_1(\mu) = N_{493}$, $P_1(\mu) = N_{492}$, $P_1(\mu) = N_{491}$, $P_1(\mu) = N_{490}$, $P_1(\mu) = N_{489}$, $P_1(\mu) = N_{498}$, $P_1(\mu) = N_{487}$. Thus, $o(P_1 = A_6)$, $o(P_1 = N_{499})$, $o(P_1 = N_{498})$, $o(P_1 = N_{497})$, $o(P1 = N_{496})$, $o(P1 = N_{495})$, $o(P1 = N_{494})$, $o(P_1 = N_{493})$, $o(P_1 = N_{492})$, $o(P_1 = N_{491})$, $o(P_1 = N_{490})$, $o(P_1 = N_{489})$, $o(P_1 = N_{488})$, $o(P_1 = N_{487}) = 1$, then we have one chain, which is $N_{499} < A_6$.

Similarly, we have one fuzzy subgroup for $P_1(\mu) = M_{477}$, $P_1(\mu) = M_{478}$, $P_1(\mu) = M_{479}$, $P_1(\mu) = M_{480}$, $P_1(\mu) = M_{481}$, $P_1(\mu) = M_{482}$, $P_1(\mu) = M_{483}$, $P_1(\mu) = M_{484}$, $P_1(\mu) = M_{485}$, and $P_1(\mu) = M_{486}$, then we have one chain for each, Similarly,
we have one fuzzy subgroup for $P_1(\mu) = L_{447}, \cdots, P_1(\mu) = L_{474}$, $P_1(\mu) = L_{475}$, $P_1(\mu) = L_{475}$, then we have one chain for each. Similarly if $P_1(\mu) = J_{436}$, then we have four chains, those are $J_{436} < A_6$ and $J_{436} < N_{499} < A_6$, $J_{436} < N_{498} < A_6$, $J_{436} < L_{476} < A_6$. Therefore, we get four fuzzy subgroups of A_6. We also have four fuzzy subgroups for $P_1(\mu) = J_{407}, \cdots, P_1(\mu) = J434$, $P1(\mu) = J435$, $P1(\mu) = J436$.
For $P_1(\mu) = K_{437}$, we have two chains, those are $K_{437} < A_6$ and $K_{437} < M_{477} < A_6$. Therefore, we get two fuzzy subgroups of A_6 with $P_1 = K_{437}$, those are

$$\mu_{94}(x) = \begin{cases} \theta_1, & x \in K_{437} \\ \theta_2, & x \in A_6 \setminus K_{437} \end{cases}, \quad \mu_{94}(x) = \begin{cases} \theta_1, & x \in K_{437} \\ \theta_2, & x \in M_{477} \setminus K_{437} \\ \theta_3, & x \in A_6 \setminus K_{437} \end{cases} \quad (4)$$

For $P_1(\mu) = K_{438}, \cdots, P_1(\mu) = K_{444}$, $P_1(\mu) = K_{445}$, $P_1(\mu) = K_{446}$. We have two fuzzy subgroups.

If $P_1(\mu) = I_{406}$, then we have three chains, those are $I_{406} < A_6$ and $I_{406} < N_{499} < A_6$, $I_{406} < N_{498} < A_6$. Therefore, we get three fuzzy subgroups of A_6 with $P_1 = I_{406}$.

We have three fuzzy subgroups if we have $P_1(\mu) = I_{371}, \cdots, P_1(\mu) = I_{404}, P_1(\mu) = I_{405}$.
By similar method, we have;

1. Four fuzzy subgroups of A_6, if we have $P_1(\mu) = H_{361}, \cdots, P_1(\mu) = H_{368}$, $P_1(\mu) = H_{389}$, $P1(\mu) = H_{370}$.
2. Three fuzzy subgroups A_6, if we have $P_1(\mu) = G_{316}, \cdots, P_1(\mu) = G_{358}$, $P_1(\mu) = G_{359}$, $P1(\mu) = G_{360}$.
3. Ninetheen fuzzy subgroups A_6, if we have $P_1(\mu) = F_{196}, \cdots, P_1(\mu) = F_{313}$, $P_1(\mu) = F_{314}$, $P_1(\mu) = F_{315}$.
4. Six fuzzy subgroups A_6, if we have $P_1(\mu) = E_{160}, \cdots, P_1(\mu) = E_{193}$, $P_1(\mu) = E_{194}$, $P_1(\mu) = E_{195}$.
5. Six fuzzy subgroups A_6, if we have $P_1(\mu) = D_{85}, \cdots, P_1(\mu) = D_{127}$, $P_1(\mu) = D_{128}$, $P_1(\mu) = D_{129}$.
6. Eighteen fuzzy subgroups A_6, if we have $P_1(\mu) = V_1$, $P_1(\mu) = V_2$, $P_1(\mu) = V_3, \cdots$, $P_1(\mu) = V_{30}$
7. For fuzzy subgroups A_6, if we have $P_1(\mu) = C_{45}, \cdots, P_1(\mu) = C_{82}$, $P_1(\mu) = C83$, $P_1(\mu) = C_{84}$.
8. For fuzzy subgroups A_6, if we have $P_1(\mu) = B_1, \cdots, P_1(\mu) = B_{42}$,

$P_1(\mu) = B_{43}, \quad P_1(\mu) = B_{44}$.

Thus, the total number of fuzzy subgroups of A_6 is 30548.

4. Conclusion

This work has extended the research on counting distinct fuzzy subgroups of alternating groups to alternating group of degree 6. The altimate goal in this direction is to establish an explicit formula for counting distinct fuzzy subgroups of A_n however this result serves as a resource document for researchers on fuzzy algebraic structures of alternating groups and also modified the counting of distinct fuzzy subgroups of A_5 . Furthermore, we obtained 30548 distinct fuzzy subgroups of A_6 .

References

Ogiugo M.E and EniOluwafe M. (2017), Classifying a class of the fuzzy subgroups of the alternating groups An, *African Journal of Pure and Applied Mathematics,* Vol. 4. No. 1, pg.27-33.l Classifying a class of the fuzzy subgroups of the

Priyanka K.Neeraj M. and Parvinder S. (2010), Relevance of Fuzzy Concept in Mathematics, *International Journal of Innovation, Management and Technology*. Vol. 1. No.3.

Sulaiman R. and ABD Ghafur A. (2012), Constructing Fuzzy Subgroups of Symmetric groups S4*, International Journal of Algebra*. Vol. 6. No.1. Pg. 23-28.

Sulaiman R. and ABD Ghafur A. (2011), Counting Fuzzy Subgroups of Symmetric Groups S2; S3; and Alternating Group A4: *Journal of Quality Measurement and Analysis* Vol. 6(1). Pg. 57-63.