MOMENTUM DISTRIBUTION CALCULATIONS OF ^{28}Si CORE FRAGMENT FROM ^{29}P + ^{12}C AND ^{12}C CORE FRAGMENT FROM ^{13}C + ^{9}Be REACTIONS USING GLAUBER THEORY

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Abstract

*Momentum distribution calculations are important to investigate signatures of somehalo nucleus. In this paper, the results of the calculated longitudinal momentum distribution of fragments from the reaction systems of^{29}P + ^{12}C and ^{13}C + ^{9}Be computed in the framework of Glauber theory using the Computer Science Communication Glauber Model (CSC_GM) program code was presented, which provides the first order internal momentum distribution of removed nucleons. The CSC_GM code is a Fortran 90 program code, which was run on Linux operating system. The projectile nucleus is assumed to have the structure of a core plus a valence nucleon. The result showed that the narrow momentum distribution of ^{28}**Si** fragment showed a proton-halo structure for the nuclei and are found to agree with the experimental data. Also the result for ^{12}**C** nucleus with a broad momentum distribution showed no any sign of halo structure.*

Keywords: Glauber model; One-nucleon halo; Momentum distribution

Introduction

The momentum distribution of a fragment is one of the quantities measured in the experimental study of unstable radioactive nuclei (Adamu, 2013a; Ogawa, 1997). Other relevant quantities also measured in this type of study are the various reaction cross-sections that include the total reaction cross-section, nucleon removal cross-section, etc. These quantities play important role in revealing the nuclear structure of unstable nuclei, particularly the structures of halo and skin (Shukla et al. 2004). Halo and skin are exotic nuclear properties or structures that are peculiar to only unstable radioactive nuclei. Sizes and density distributions (of both nuclear matter and charge) of unstable nuclei are therefore quite different from those of stable nuclei (Ozawa et al. 2001). Measurement of longitudinal momentum distributions of ^{28}Si fragment arising from $^{29}P+^{12}C$ reaction indicates the spatially extended density distributions of the loosely bound proton in ^{29}P (Yi-Bin et al. 2005). The CSC_GM program code is employed to compute the various cross sections that reveal some nuclear structures. Particular interesting cases are ^{29}P and ^{13}C. The quantity measured in this study is the momentum distributions of fragment for the reactions of $^{29}P+^{12}C$ and $^{13}C+^{9}Be$ systems which are calculated in the framework of the Glauber theory using the CSC_GM program code. The nucleus ^{29}P for example, has an extremely low binding energy for the valence proton and is considered to be a proton-halo candidate.

According to Heisenberg's uncertainty principle, a narrow momentum distribution implies a large spatial extent of the valence nucleon's wave function (Cortina-Gil et al. 2001). Tostevin et al.(2002) explains these features of semi-classical approximations as those that are implicitly energy non-conserving and the calculations do not treat energy shearing between the center of mass and relative motion degrees of freedom of the neutron and core or the momentum transfers involved in the deflection of the core from its assumed (eikonal) straight line path. As a result, the calculated distributions must be symmetric about the momentum distribution corresponding to the beam velocity and the experimental asymmetry pronounced for the halo states suggests that the phenomenon is associated with the elastic breakup mechanism and that there is a need to go beyond the eikonal theory. The Glauber theory is a microscopic reaction theory of high-energy collision based on the eikonal approximation and on the bare nucleon–nucleon interaction. It is now a standard tool to calculate the momentum distributions because it can account for a significant part of breakup effects which play an important role in the reaction of a weakly bound nucleus (Glauber et al. 1959).

Methodology

Considering the reaction between a projectile nucleus P with a target nucleus T, at the first stage of the reaction, the projectile nucleus in the ground state is described with an intrinsic wave function Ψ_0 which impinges with momentum $\hbar\boldsymbol{k}$ = (0,0, $\hbar\boldsymbol{k}$) on the target in its ground state, described with an intrinsic wave function θ_o . The centre-of-mass wave function is removed from Ψ_0 (θ_o). At the final stage of the reaction, the projectile goes to state a specified by a wave function Ψ_a and the target goes to another state c specified by a wave function θ_c. The state a may be a continuum state that includes some fragments. The momentum transferred from the target to the projectile is $\hbar\boldsymbol{q}$ (Abu-Ibrahim and Suzuki, 2002). Tanihata(1996) defined the scattering amplitude for this reaction written in the Glauber theory as:

$$F_{ac}(q) = \frac{i\boldsymbol{k}}{2\pi}\int db e^{-iq.b}\langle\Psi_a\Theta_c|1 - \prod_{i\in P}\prod_{j\in T}(1 - \Gamma_{ij})\,|\Psi_o\Theta_o\rangle \tag{1}$$

The integrated cross section for this reaction is given by:

$$\sigma_{ac} = \int \frac{dq}{\boldsymbol{k}^2}\,|F_{ac}(q)|^2, \tag{2}$$

where b is the impact parameter between the projectile and the target.

The profile function Γ in Eqn. (1) is given by:

$$\Gamma_{(b)} = \frac{1-i\alpha}{4\pi\beta}\,\sigma_{NNe^{-b^2/2\beta}}. \tag{3}$$

The parameters σ_{NN}, α, and β usually depend on either the proton-proton (neutron-neutron) or proton-neutron case. The argument of Γ_{ij} in Eqn. (1) is **b** + $\mathbf{s}^P$ – $\mathbf{s}^T$, which stands for the impact parameter between i^{th} and j^{th} nucleons. Here $\mathbf{s}^P$ ($\mathbf{s}^T$) is the two-dimensional coordinates which comprises the x- and y-components of the ith nucleon coordinate in the projectile (target) relative to its Centre-of-mass coordinate.

The momentum distribution of the core fragment is therefore calculated here after the inelastic breakup of the projectile. The one-nucleon removal reaction is contributed by both the elastic and inelastic process with the inelastic process becoming dominant at high energies beyond a few hundred MeV/nucleon (Basdevant et al. 2005). Assuming the momentum of the core $P = (P_{\perp}, P_{\|})$ and that of the nucleon going to the continuum state be ħ**k**. Assuming that the core will remain in its ground state, the momentum distribution is calculated by Equation (4) (Adamu, 2013b).

$$\frac{d\sigma_{-N}^{inel}}{dp} = \int \frac{dq}{k^2} \sum_{c\neq 0} \int dk\delta\left(p - \frac{A_C}{A_P}\hbar q + \hbar \boldsymbol{k}\right) |F_{(k,0)c}(q)|^2 \tag{4}$$

Since the momentum transfer received by the ejected valence nucleon will be considered to be large, the final state interaction can be ignored. The continuum scattering wave function of the last nucleon is then approximated by a plane wave:

$$\varphi(r) = \frac{1}{(2\pi)^{2/3}} e^{-p.r}, \tag{5}$$

and Equation (4) then becomes:

$$\frac{d\sigma_{-N}^{inel}}{dp} = \int db_N \{1 - e^{-2lm} \chi_{NT}(b_N)\} \times$$
$$\frac{1}{(2\pi\hbar)^3} \frac{1}{2j+1} \sum_{mm_s} |\int dr e^{\frac{i}{\hbar}p.r} \chi^*_{\frac{1}{2}m_s} e^{i\chi_{CT}(b_N - s)} \varphi_{nljm}(r)|^2, \tag{6}$$

where b_N stands for the impact parameter of the balance nucleon with respect to the target, $\varphi_{nljm(r)}$ is the valence nucleon wave function. The phase-shift functions of the core-target and nucleon-target χ_{CT}, χ_{NT} respectively will be defined through the relevant densities (ρ) by (Adamu, 2013a; Ogawa et al. 2003).

$$\int i\chi_{CT}(b) = -\int dr \int dr' \rho_C(r) \rho_T(r')\, \Gamma(\boldsymbol{b} + \boldsymbol{s} - \boldsymbol{s}'), \tag{7}$$
$$\int i\chi_{NT}(b) = -\int dr \rho_T(r)\, \Gamma(\boldsymbol{b} - \boldsymbol{s}). \tag{8}$$

Then the density (ρ) is given by:

$$\rho_{(r)} = \sum_i c_i e^{-a_i r^2}, \tag{9}$$

and is normalized to the mass number A of a nucleus as:

$$A = \int \rho(r) dr \quad . \tag{10}$$

Then the integral of Equation (6) over transverse momentum will leads to the longitudinal momentum distribution (Ogawa et al. 2003).

$$\frac{d\sigma_{-N}^{inel}}{dp_{\|}} = \int dp_{\lrcorner} \frac{d\sigma_{-N}^{inel}}{dp} = \frac{1}{2\pi\hbar} \int db_N \left(1 - e^{-2ml} \chi_{NT}(b_N)\right) \int ds \left(1 - e^{-2ml} \chi_{NT}(b_N - s)\right) \times$$
$$\int dz \int dz' e^{\frac{i}{\hbar}} P_{\|}(z - z') u^*_{nlj}(r) \frac{1}{4\pi} P_l(\widehat{r'}.\widehat{r}), \tag{11}$$

where $r = (s, z)$ and $r' = (s, z')$ and P_l is the Legendre polynomial, U_{nlj} (r) is the single-particle wave function. Then integrating Equation (11) over the Legendre polynomial P_l will yield σ_{-N}^{inel} .

The longitudinal momentum distribution is expressed as the sum of contributions from the azimuthal components of the valence-nucleon wave function (Ogawa et al. 2003). That is

$$\frac{\sigma_{-N}^{inel}}{dp_{||}} = \sum_{m_l=-l}^{l} \left(\frac{d\sigma_{-N}^{inel}}{dp_{||}}\right) m_l, \tag{12}$$

with

$$\left(\frac{d\sigma_{-N}^{inel}}{dp_{||}}\right) m_l = \frac{1}{2\pi\hbar} \int db_N \{1 - e^{-2ml}\chi_{NT}(b_N)\} \int ds\{1 - e^{-2ml}\chi_{NT}(b_N - s)\} \times \frac{1}{2l+1} \left| \int dz e^{\frac{i}{\hbar}} P_{||}(z) u_{nlj}^*(r) Y_{m_l}(\hat{r}) \right|^2. \tag{13}$$

The momentum distribution of the core will come out along the beam direction if $P_{\perp}$=0 in Equation (6) (Ogawa et al. 2003).

$$\sum_{m_l=-i}^{i} \frac{d\sigma_{-N}^{inel}}{dP} |_{P_{\perp=0}} = \frac{1}{(2\pi\hbar)^3} \frac{1}{2l+1} \int db_N \left(1 - e^{-2lm}\chi_{NT}(b_N)\right) \times \left| \iint dr\, e^{\frac{i}{\hbar}} z + i\chi_{CT}(b_N - s)\, u_{nlj} Y_{lm_l}(\hat{r}) \right|^2$$

$$\sum_{m_l=-i}^{i} \frac{d\sigma_{-N}^{inel}}{dP} |_{P_{\perp=0}} = \frac{1}{(2\pi\hbar)^3} \int db_N \left(1 - e^{-2lm}\chi_{NT}(b_N)\right) \times \int dr' \int dr\, e^{\frac{i}{\hbar}} P_{||}(z - z') + \quad i\chi_{CT}(b_N - s) - i\chi_{CT}^*(b_N - s') u_{nlj}^*(r') u_{nlj}(r) \frac{1}{4\pi} P_l(\widehat{r'}.\hat{r}), \tag{14}$$

where $r = (s, z)$ and $r' = (s', z')$.

The CSC_GM code is a Fortran 90 program capable of calculating various types of reactions cross sections. It was used to calculate the longitudinal momentum distributions of various reactions consisting of a core plus one valence-nucleon system in the framework of the Glauber model after inelastic break-up. The input file csc.inpcontains data to be read into the main program at run-time. The output file momdist.outkeeps the results of the computation. The code was modified and run in the digital *Linux* operating system (Ubuntu version 14.04). The input data for the calculation of momentum distributions of ^{28}Si from the two reaction systems (a) and (b) are represented in Tables 1A and 2A

$$^{29}P + ^{12}C \rightarrow ^{28}Si + ^{12}C + p \tag{a}$$

$$^{13}C + ^{9}Be \rightarrow ^{12}C + ^{9}Be + n \tag{b}$$

The input data contained in the file csc.inp specify the reaction system, the momentum distribution, the target and core densities, and the control parameters of the Metropolis algorithm etc. The first line gives the mass numbers of the target, projectile and core (A_T, A_P, and A_C), the second line gives the charge numbers of those nuclei (Z_T, Z_P, and Z_C). The code assumes $A_P - A_C = 1$. The third line defines the incident energy of the projectile per nucleon (in MeV). The fourth line defines the parameters of the nucleon–nucleon profile function, as used in Equation (7): σ_{NN} (in fm^2),$\propto$, and β (in fm^2). For the zero-range profile function, β=

0.0. The fifth line gives the orbital angular momentum of the valence nucleon. The sixth line specifies the condition for the Monte Carlo quadrature, the number of configuration points (N_s), the step size (δ in fm) for the random walk in the Metropolis algorithm, and the seed number for generating random numbers (irand). The seventh line gives the number of Gaussians used to fit the core and target densities. The eighth line gives the coefficients $c_i$and the ranges a_i(in fm^{-2}) of the target density. The ninth line gives coefficients $c_i$and the ranges a_i(in fm^{-2}) of the core density as defined by Equation (9). The last line defines the maximum angle (in degrees) to be calculated. Results of the calculations of the momentum distributions of the core fragment from Equation (14) are written on the filemomdist.out in Tables 1C and 2C for the two reaction systems. The single-particle wave function of the valence nucleon is generated by the code by specifying quantum numbers of the wave function in the input file wf.inpin Table 1B**.** To obtain the radial part of the single-particle wave function, $R_{nlj}(r) = ru_{nlj}(r)$ which is used as the guiding function w(x) we solve a Schrödinger equation with a potential $U(r)$ (Ogawa et al. 2003):

$$\frac{d^2R(r)}{dr^2} + \frac{2\mu}{\hbar^2}\left[E - U(r) - \frac{l(l+1)\hbar^2}{2\mu r^2}\right]R(r) = 0 \quad (15)$$

$$U(r) = -V_0 f(r) + V_{ls}(l.s){r_0}^2 \frac{1}{r}\frac{d}{dr}f(r) + V_{coul} \quad (16)$$

where

$$f(r) = \left[1 + exp\frac{r-R}{a}\right]^{-1} \quad \text{With } R = r_0 A_C^{1/3}$$

where $r_0$and $a$are the radius and diffuseness parameters in fm respectively. V_0 is the initial depth of the potential. V_{ls} =17MeV (Adamu, 2013b).

Results and Discussion

Momentum distributions for the $^{29}P+^{12}C$ system

The ^{29}P nucleus was assumed to have the structure of a ^{28}Si + proton, where the proton orbit is in $1s_{1/2}$ state with the separation energy of $\varepsilon = 2.748\, MeV$ (Yi-Bin et al. 2005). The momentum distribution was calculated at energy of 30.7 MeV/nucleon. This very binding energy explains why the ^{29}P is suspected to be a proton-halo candidate. The density of ^{28}Si was taken from (Sharma et al. 2013).

Figure 1 compares with experiment the longitudinal momentum distribution of ^{28}Si from $^{29}P+^{12}C$ reaction. The experimental data are obtained from (Yi-Bin et al. 2005). The solid curve is the result of Equation (14). The input parameters which have to be filled in the files csc.inp are shown in Table 1A.

Table 1A: csc.inp Input Files for $^{29}P+^{12}C$ Reaction System

INPUT PARAMETERS	VALUES
Mass numbers of target, projectile and core: (A_T; A_P; A_C)	12, 29, 28
Atomic numbers of target, projectile and core: (Z_T; Z_P; Z_C) 6, 15, 14	
Incident Energy per nucleon (in MeV) 30.7	
Profile function parameters (σ_{NN}, α and β in fm^2) 19.6, 0.87, 0.00	
l (angular momentum quantum number)0	
Monte Carlo parameters (Ns, δ, irand) 500000, 2.5, −11213	
Number of Gaussians used to fit the core and target densities 2	
Coefficient c_i, rangea_i(in fm^{-2}) of the Target	-1.14333 0.285974
0.28340 0.285598	

Coefficient c_i rangea_i(in fm-2) of the Core	-0.85374 0.033495
0.99676 0.059533	
Maximum angle (in degrees)30	

Table 1B:wf.inp Input file for $^{29}P+^{12}C$Reaction System

INPUT PARAMETERS	VALUES
Initial depth V_0 of the optical potential (in MeV)	44.3
Diffuseness parameter a (in fm)	0.7
Radius parameter r_0 (in fm)	1.3
Energy eigenvalue for the valence nucleon (in MeV)	-2.748
j value for the valence nucleon orbit 0.5	
Node number for the valence nucleon orbit 1	

Table 1C:momdist.out output file formatfor $^{29}P+^{12}C$Reaction System

$P_{\|\|}$[MeV/c]	dσ/dp [mb/(MeV/c)]
0.0000000000000000 1.6221302731503257	
10.0000000000000001.5729947060255777	
20.0000000000000001.4362040118754215	
30.0000000000000001.2394641571294549	
……………………..	……………………..

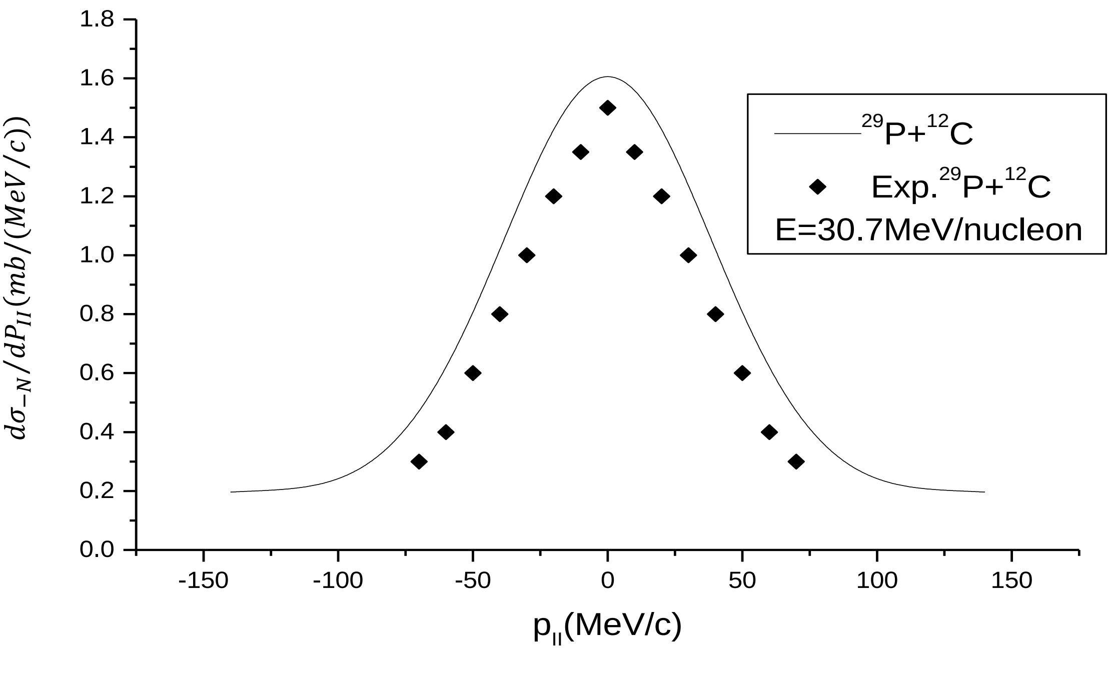

Figure1. Longitudinal Momentum Distribution of the ^{28}Si for the Reaction $^{29}P+^{12}C$ at Energy of 30.7MeV/Nucleon. The solid Curve Denotes the Results Calculated from Equation (14). The Experimental Data are taken from (Yi-Bin et al. 2005).

Momentum distributions for the ^{13}C+^{9}Be system

The ground state of ^{13}C was assumed to be ^{12}C + $0P_{1/2}$- neutron system with n = 0, l = 1, j = 0.5, and the separation energyε=4.946 MeV (Ogawa et al. 2003). The longitudinal momentum distribution of the ^{12}C for the reaction ^{13}C+^{9}Be→ ^{12}C + ^{9}Be + n at the energy of 1.1GeV/nucleon is shown in Figure 2.

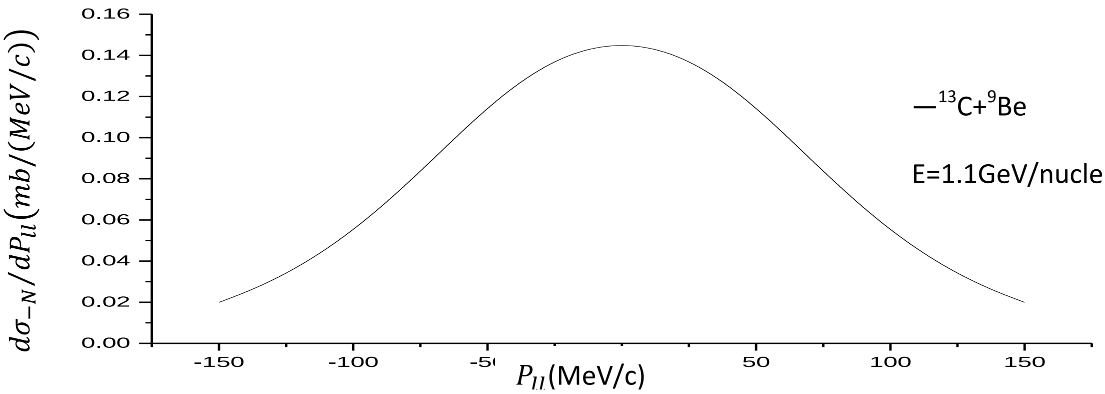

Figure 2. Longitudinal Momentum Distribution of the ^{12}C for the Reaction ^{13}C+^{9}Be at Energy of 1.1GeV/Nucleon. The Curve Denotes the Results Calculated from Equation (14).

Table 2A: **csc.inp** Input Files for ^{13}C+^{9}Be Reaction System

INPUT PARAMETERS	VALUES
Mass numbers of target, projectile and core: (A_T; A_P; A_C) 9, 13, 12	
Atomic numbers of target, projectile and core: (Z_T; Z_P; Z_C) 4, 6, 6	
Incident Energy per nucleon (in MeV) 1100	
Profile function parameters (σ_{NN}, α and β in fm²) 4.32, -0.275, 0.22	
l (angular momentum quantum number) 1	
Monte Carlo parameters (Ns, δ, irand) 500000, 2.5, −11213	
Number of Gaussians used to fit the core and target densities 2	
Coefficient c_i, rangea_i(in fm^{-2}) of the Target	
-0.06240 0.69377	
0.28340 0.30012	
Coefficient c_i rangea_i(in fm^{-2}) of the Core	
-1.14333 0.28597	
0.28340 0.285598	
Maximum angle (in degrees)30	

Table 2B: **wf.inp** Input file for^{13}C+^{9}BeReaction System

INPUT PARAMETERS	VALUES
Initial depth V_0 of the optical potential (in MeV)	44.3
Diffuseness parameter a (in fm)	0.7
Radius parameter r_0 (in fm)	1.3
Energy eigenvalue for the valence nucleon (in MeV)	-4.946
j value for the valence nucleon orbit 0.5	
Node number for the valence nucleon orbit 1	

Table 2C: **momdist.out** output file format $^{13}C+^{9}Be$ Reaction System

$P_{||}$[MeV/c]	dσ/dp [mb/(MeV/c)]
0.0000000000000000 0.14518757622360320	
10.0000000000000000.14390456775282656	
20.0000000000000000.14005586810148671	
30.0000000000000000.13368631509854914	
……………………..	…………………….

Conclusion

The ^{13}C nucleus, with a broad calculated momentum distribution shows no sign of skin or halo structure. However, the narrow momentum distributions of fragments from the ^{29}P + ^{12}C reactions indicates ^{28}Si to be a proton-halo where the proton orbit is in $1s_{1/2}$ state with separation energy of $\varepsilon = 2.748\ MeV$ (Yi-Bin et al. 2005). The agreement of these results with experiment proves the success of the Glauber Theory and makes the core plus one nucleon model of the radioactive nuclei to be valid. It also validates the CSC_GM program used in the computations.

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